2 Extensions of the Einstein's vacuum equations and their solutions

One of the aims of metraphysics is to eliminate the concept of mass as a fundamental property. We present here a promising approach to achieving this end. In order to do this, we consider the interface between different solutions of the Einstein field equations, and construct an extension of these equations and their solutions. This forms the basis of a metric-dynamic model of particles of varying sizes, including virtually all elementary particles that are part of the Standard Model.

This Chapter considers three types of Einstein's vacuum equations and the totality of their solutions. It is shown that it makes sense to consider the sums and/or of averaging various solutions of the same Einstein vacuum equation, despite the fact that these equations are nonlinear. The Chapter is aimed at the development of differential geometry and the program of Clifford-Einstein-Wheeler for the complete geometrization of physics.

A note on terminology: *New concepts are introduced using either terms coined by the author, or new usages of words already in use for similar concepts. At appropriate places in the text, we call the attention of the reader to the new terminology with explanations preceded by the word "terminology" in bold. These terms are tentative, and the author welcomes suggestions for improvements on the terminology.*

2.1 The first Einstein's vacuum equations and its solutions

1(a) The Einstein - Hilbert equation for vacuum has the form

$$R_{ik} - \frac{1}{2} R g_{ik} = 0, \qquad (2.1.1)$$

where g_{ij} are metric tensor components;

$$R_{ik} = \frac{\partial \Gamma^{l}_{ik}}{\partial x^{l}} - \frac{\partial \Gamma^{l}_{il}}{\partial x^{k}} + \Gamma^{l}_{ik}\Gamma^{m}_{lm} - \Gamma^{m}_{il}\Gamma^{l}_{mk} \quad \text{is the Ricci tensor;} \qquad (2.1.2)$$

$R = g^{ik}R_{ik}$ represents scalar curvature; (2.1.3)

$$\Gamma^{\lambda}_{ik} = \frac{1}{2} g^{\lambda\mu}\left(\frac{\partial g_{\mu k}}{\partial x^{i}} + \frac{\partial g_{i\mu}}{\partial x^{k}} - \frac{\partial g_{ik}}{\partial x^{\mu}}\right) \quad \text{are Christoffel symbols.} \qquad (2.1.4)$$

Combining (2.1.1) with g_{ik}, we obtain [38]

$$g^{ik}\left(R_{ik} - \frac{1}{2} R g_{ik}\right) = R - \frac{n}{2} R = 0, \qquad (2.1.5)$$

because $g^{ik}g_{ik} = n$ of the number of spatial dimensions.

For any n-dimensional space (except for $n = 2$), Equation (2.1.5) can only be performed when $R = 0$. Therefore, for $n = 4$, Equation (2.1.1) becomes

$$R_{ik} = 0 \,. \qquad (2.1.6)$$

Expression (2.1.6) will be called the first Einstein's vacuum equation.

The solutions to (2.1.6) are best expressed, as a rule, in a spherical coordinate system in the form of metrics. Before we present these metrics, we need to insert a note about our terminology.

1(b) **Terminology** (the following italicized notes are due to the translator): *The term "signature" used here is an extension of the usual means to determine where a metric component is positive definite or negative definite. More broadly, suppose a space S of points* $s=(x_0,x_1,x_2,x_3)$ *has several metrics or pseudometrics defined on it, such that each metric or pseudometric* $[ds]_i$ *is described by*

$$[ds]_i^2 = a_0 f_1(s,p)dx_0^2 + a_1 f_2(s,p)dx_1^2 + a_2 f_3(s,p)\, dx_2^2 + a_3 f_3(s,p)dx_3^2,$$

where for $i \in \{0,1,2,3\}$, $f_i(s)$ *are positive definite functions defined on S, p are given parameters, and* $a_i \in \{0,-1,1\}$. (*For convenience, we shall drop "or pseudometric" and the mention of the parameters in the rest of this section.*) *Then, if this is a quadratic (metric) form, we form the ordered tuple* (a_0, a_1, a_2, a_3), *whereby −1 is abbreviated "−" and 1 is abbreviated "+" (0 retains its name). We then term it a "signature" of the metric. If, on the other hand, the defining equation of each of the metrics is a linear (affine) form or "colored" quaternion ("colored" to be explained later), we term it a "stignature" to emphasize this difference. However, in what follows, the rules for signatures extend in a natural way to stignatures.*

Suppose further that several metrics are defined on the region in question such that they only differ in the sign of their coefficients. This would allow a set of 64 possible metrics in such a set.

Now we use the fact that the sum of two metrics yields another metric. To complement this situation, we can define an operation, a component-wise addition: if there are two signatures in the set (a_0, a_1, a_2, a_3) *and* (b_0, b_1, b_2, b_3), *then* (a_0, a_1, a_2, a_3) $\tilde{\mp}$ (b_0, b_1, b_2, b_3) = (a_0+b_0, a_1+b_1, a_2+b_2, a_3+b_3) *if and only if* (a_0+b_0, a_1+b_1, a_2+b_2, a_3+b_3); *that is, the sum of the signatures of metrics is the signature of the sum of the corresponding metrics. Such a set of 64 signatures will form a group under* $\tilde{\mp}$ *.We henceforth drop the tilde, using + for both normal addition and this operation, where the difference will be clear from the context.*

We can also form various substructures. For example, the aforementioned difference between a signature and a stignature is one distinction. Restrictions of the fact that the metrics are defined on spacetime introduces further restrictions. Furthering such considerations, the functions we will be using will fulfill the condition that $a_0 \times a_1 \times a_2 \times a_3 = 0$ *if and only if* $a_0 = a_1 = a_2 = a_3 = 0$. *(The reason for this will become apparent later in the paper. Since the resulting substructure of only 17 elements no longer forms a group under the same operation as before, lacking closure, further restrictions on the opera-*

tion needs to be made to enjoy the consequences of the group structure.) Other restrictions will limit the number of elements even further, or require further structure. Taking all of these possibilities together into a single structure is beyond the scope of this paper, but this algebra, which we term the "Algebra of Signatures".

In this paper, most of the metrics will be expressed so that the spatial portion is expressed in spherical coordinates (r, θ, φ), so that $x_0 \equiv t$, $x_1 \equiv r$, $x_2 \equiv \theta$, $x_3 \equiv \varphi$, and the metric is expressed as:

$$ds^2 = a_1 f_1(t)dt^2 + a_2 f_2(r)dr^2 + a_3 f_3(r)\, d\theta^2 + a_4 f_4(r,\theta)d\varphi^2.$$

For this reason, we shall refer to the regions of vacuum on which the metrics are defined as "spherical formations". The fact that measurements of most particles are spherically symmetrical is a further support for the intuitive feel of this term.

Solutions of equation (2.1.6) are considered in many works on modern differential geometry and general relativity. However, in none of the publications known to the author, the relationship between the various solutions of this equation is discussed, so we will consider it in sufficient detail.

Solutions of equation (1.6) are usually sought in a spherical coordinate system in the form of metrics:

$$ds^{(-)2} = e^{\nu}c^2dt^2 - e^{\lambda}dr^2 - r^2d\theta^2 - r^2\sin^2\theta\, d\varphi^2 \text{ with the signature } (+---), \qquad (2.1.7)$$

$$ds^{(+)2} = -e^{\nu}c^2dt^2 + e^{\lambda}dr^2 + r^2d\theta^2 + r^2\sin^2\theta\, d\varphi^2 \text{ with the signature } (-+++), \qquad (2.1.8)$$

where ν and λ are the sought-after functions of t and r respectively.

As a result of the substitution of covariant and contravariant components of the metric tensor of the metric (2.1.7) in equation (2.1.6) for fixed (i.e., time-independent) vacuum states, we obtain a system of three equations [34]:

$$\nu = -\lambda; \qquad (2.1.9)$$

$$-e^{\nu}(\nu'/r + 1/r^2) + 1/r^2 = 0; \qquad (2.1.10)$$

$$\nu'' + \nu'^2 + 2\nu'/r = 0. \qquad (2.1.11)$$

The differential equation (2.1.10) has three solutions:

$$\nu_1 = \ln(h_1 + h_2/r), \qquad \nu_2 = \ln(h_1 - h_2/r), \qquad \nu_3 = h_3, \qquad (2.1.12)$$

where h_1, h_2, h_3 are integration constants.

Equation (2.1.11) also has three solutions:

$$\nu_1 = \ln(1 + b/r), \qquad \nu_2 = \ln(1 - b/r), \qquad \nu_3 = 0, \qquad (2.1.13)$$

where b is a constant of integration.

If $h_1 = 1$, $h_2 = b$, and $h_3 = 0$, the solutions to (2.1.12) and to (2.1.13) coincide.

Substituting the three possible solutions (2.1.13) in the metric (2.1.7) with (2.1.9) we obtain the three metrics with the same signature (+ – – –):

$$ds_a^{(-)2} = (1-r_0/r)c^2dt^2 - (1-r_0/r)^{-1}dr^2 - r^2d\theta^2 - r^2\sin^2\theta\, d\varphi^2, \qquad (2.1.14)$$

$$ds_b^{(-)2} = (1+r_0/r)c^2dt^2 - (1+r_0/r)^{-1}dr^2 - r^2d\theta^2 - r^2\sin^2\theta\, d\varphi^2, \qquad (2.1.15)$$

$$ds_c^{(-)2} = c^2dt^2 - dr^2 - r^2d\theta^2 - r^2\sin^2\theta\, d\varphi^2. \qquad (2.1.16)$$

where $r_0 = b$ is the radius of the corresponding closed sphere.

By doing the same operations with the components of the metric tensor of the metric (2.1.8), we obtain the following three metrics, also satisfying Equation (2.1.6), but with opposite signature $(- + + +)$:

$$ds_a^{(+)2} = -(1-r_0/r)c^2dt^2 + (1-r_0/r)^{-1}dr^2 + r^2d\theta^2 + r^2\sin^2\theta\, d\varphi^2, \qquad (2.1.17)$$

$$ds_b^{(+)2} = -(1+r_0/r)c^2dt^2 + (1+r_0/r)^{-1}dr^2 + r^2d\theta^2 + r^2\sin^2\theta\, d\varphi^2, \qquad (2.1.18)$$

$$ds_c^{(+)2} = -c^2dt^2 + dr^2 + r^2d\theta^2 + r^2\sin^2\theta\, d\varphi^2. \qquad (2.1.19)$$

Each of the metrics (2.1.14) through (2.1.19) is irreducible to the others; together this is called a generalized Schwarzschild metric.

Metrics (2.1.14) through (2.1.19) describe the state of the same region of the vacuum. Therefore we consider different variants of their averages, in spite of the fact that equation (2.1.6) is non-linear; in general, in such cases the sum of the solutions is not itself a solution.

If the centers of the metrics (2.1.14) through (2.1.16) and (2.1.17) through (2.1.19) coincide, evidently they will sum to zero

$$ds_a^{(-)2}+ds_b^{(-)2}+ds_c^{(-)2}+ds_a^{(+)2}+ds_b^{(+)2}+ds_c^{(+)2} = 0\cdot c^2dt^2 + 0\cdot dr^2 + 0\cdot d\theta^2 + 0\cdot\sin^2\theta\, d\varphi^2 = 0. \qquad (2.1.20)$$

The resulting metric is

$$ds^{(0)2} = g_{ij}^{(0)}dx^i dx^j, \qquad (2.1.21)$$

where

$$g_{ij}^{(0)} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \qquad (2.1.22)$$

which is also a trivial solution of the vacuum equation (2.1.6).

Thus, contrary to expectation, the addition of the six metrics (2.1.14) through (2.1.19) leads to the production of additional solutions of (2.1.6).

Let us now consider the arithmetic average of the two metrics (2.1.14) and (2.1.15)

$$ds_{ab}^{(-)2} = \tfrac{1}{2}(ds_a^{(-)2} + ds_b^{(-)2}) = c^2dt - \frac{r^2}{r^2 - r_0^2}dr^2 - r^2d\theta^2 - r^2\sin^2\theta d\varphi^2 \qquad (2.1.23)$$

The distance between two points r_1 and r_2 in a region with signature $(+ - - -)$ is determined by the following expression in General Relativity:

$$r_2 - r_1 = \int_{r_1}^{r_2} \sqrt{-g_{11}^{(-)}}\, dr \,. \qquad (2.1.24)$$

By substituting $g_{11}^{(-)}$ into the average of the metric (2.1.23), we obtain

$$r_2 - r_1 = \int_{r_1}^{r_2} \sqrt{-(-\frac{r^2}{r^2 - r_0^2})}\, dr = \int_{r_1}^{r_2} \frac{rdr}{\sqrt{r^2 - r_0^2}} = \sqrt{r^2 - r_0^2}\,\Big|_{r_1}^{r_2} \,. \qquad (2.1.25)$$

First we find the value of the interval between the points $r_1 = 0$ and $r_2 = r_0$:

$$\sqrt{r^2 - r_0^2}\,\Big|_0^{r_0} = -\sqrt{-r_0^2} = -\sqrt{-1}r_0 = -ir_0 \,. \qquad (2.1.26)$$

The length of this segment is equal to the radius of the cavity r_0, and the imagination of this result suggests that equation (2.1.6) does not describe the closed region of the vacuum (spherical cavity) with radius r_0.

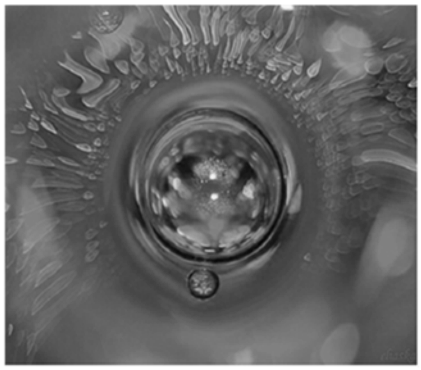


Fig. 2.1.1 Air bubble in liquid

Outside this cavity, i.e. from $r_1 = r_0$ to $r_2 = \infty$, we have

$$r_2 - r_1 = \sqrt{r^2 - r_0^2}\,\Big|_{r_0}^{\infty} = \sqrt{\infty^2 - r_0^2} \,. \qquad (2.1.27)$$

In the absence of deformation, the distance between points $r_2 = \infty$ and $r_1 = r_0$ is equal to $\infty - r_0$, and in this case this is equal to (2.1.27). The difference between these segments is approximately equal to

$$\sqrt{\infty^2 - r_0^2} - (\infty - r_0) \approx r_0 \,. \qquad (2.1.28)$$

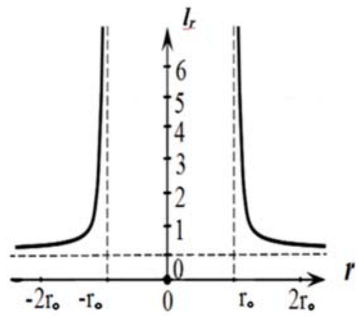


Fig. 2.1.2 Graph of the function $l_r^{(-)}$: relative length of the vacuum in the outer shell surrounding the spherical cavity. Executed in MathCad 14 for $r_0 = 2$

This result shows that the average length of the vacuum on the interval $]r_0, \infty[$ is compressed by an amount $\sim r_0$ in all radial directions due to the fact that it was forced out of the cavity radius (2.1.28). This result is similar to the air bubble in the liquid (Figure 2.1.1).

The difference between the original uncurved local area vacuum state and its current (curved) status is determined by the difference [41]

$$ds^{(-)2} - ds^{0(-)2} = (g_{ij}^{(-)} - g_{ij}^{0(-)})\, dx^i dx^j \,, \qquad (2.1.29)$$

where $ds^{0(-)2}$ – metric of the uncurved area of the vacuum;

$g_{ii}^{0(-)}$ – components of the metric tensor in the uncurved area of the vacuum.

The relative lengthening of the one of side of the vacuum region is expressed by

$$l^{(-)} = \frac{ds^{(-)} - ds^{0(-)}}{ds^{0(-)}} = \frac{ds^{(-)}}{ds^{0(-)}} - 1 \qquad (2.1.30)$$

whence it follows [41] $$ds^{(-)2} = (1 + l^{(-)})^2\, ds^{0(-)2}, \qquad (2.1.31)$$

and

$$l_i^{(-)} = \sqrt{1 + \frac{g_{ii}^{(-)} - g_{ii}^{0(-)}}{g_{ii}^{0(-)}}} - 1 = \sqrt{\frac{g_{ii}^{(-)}}{g_{ii}^{0(-)}}} - 1. \qquad (2.1.32)$$

The uncurved state of the section under consideration in a vacuum state is given by the metric (2.1.16). Therefore, substituting components $g_{ii}^{0(-)}$ and $g_{ii}^{(-)}$, respectively, from (2.1.16) and (2.1.23) to (2.1.32), we obtain

The uncurved state of the considered vacuum section is defined by the metric (2.1.16), therefore, substituting components $g_{ii}^{0(-)}$ and $g_{ii}^{(-)}$, respectively, from (2.1.16) and (2.1.23) to (2.1.32), we obtain the relative elongation of the vacuum in each radial direction in the region of r_0 to ∞

$$l_t^{(-)} = 0\,, \quad l_r^{(-)} = \frac{\Delta r}{r} - 1 = \sqrt{\frac{r^2}{r^2 - r_0^2}} - 1\,, \quad l_\theta^{(-)} = 0\,, \quad l_\varphi^{(-)} = 0\,. \qquad (2.1.33)$$

The graph of the functions $l_r^{(-)}$ is shown in Figure 2.1.2. At $r = r_0$, the function tends to infinity, and when $r < r_0$ it becomes the complex function. This once again confirms that, within the sphere of $[0, r_0]$, there is a cavity, as in Figures 2.1.1 and 2.1.2.

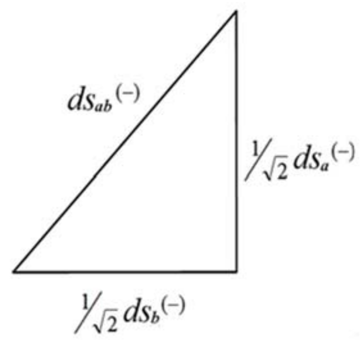


Fig. 2.1.3 Values of segments $ds_a^{(-)}$ and $ds_b^{(-)}$

Here we will not discuss the question: - What is inside a cavity with radius r_0 if there is no vacuum? Further, when considering the second Einstein vacuum equation, this problem will be solved by itself.

Thus, averaging the metrics (2.1.14) and (2.1.15) leads to the metric-dynamic description of the stable formation of a vacuum-type "air bubble in a liquid", while the metrics (2.1.14) or (2.1.15) alone do not lead to such results.

Fig. 2.1.4. A double helix can be projected onto a plane such that the tangents of the resulting curves are perpendicular to one anoth-

We note the following important fact. The average quadratic form (2.1.23)

$$ds_{ab}^{(-)2} = \tfrac{1}{2}\,(ds_a^{(-)2} + ds_b^{(-)2}) \qquad (2.1.34)$$

naturally evokes the Pythagorean theorem $a^2 + b^2 = c^2$. This means that the line segments $(\tfrac{1}{2})^{1/2} ds_a^{(-)}$ and $(\tfrac{1}{2})^{1/2} ds_b^{(-)}$ are always mutually perpendicular with respect to each other $ds_a^{(-)} \perp ds_b^{(-)}$ (Figure 2.1.3). To illustrate, a double helix can be projected onto a plane such that the tangents of the resulting curves are perpendicular to one another at the points of intersection (as shown in the typical simplified diagram of a DNA double helix in Figure 2.1.4). That is, projecting the two curves $\{(x,y,z)$: $x = r\cdot\cos t$, $y = r\cdot\sin t$, $z = kt\}$ and $\{(x,y,z)$: $x = r\cdot\cos t$, $y = r\cdot\sin t$, $z = k(t + \pi)\}$ onto the x-z plane, the tangents where the resulting plane curves meet at $z = 0$ are perpendicular to one another). By symmetry, this applies to all planes containing the z axis.

Thus, the averaged metric (2.1.23) corresponds to the segment of the "braid" consisting of four twisted “threads” (i.e. linear forms) $ds_i^{(-)}$, which form a system of two complex conjugate numbers whose product is equal to (2.1.34).

$$ds_{ab}{}^{(-)} = 1/\sqrt{2}\,(ds_a{}^{(-)} + \mathbf{i}ds_b{}^{(-)}), \qquad (2.1.35)$$

whose product is equal to (2.1.34).

In connection with the foregoing, we will call averaged metrics “k-braid” (where k is the number of averaged metrics). In particular, the averaged metric (2.1.23) is called a “2- braid”.

In connection with the above, we will call the averaged metric a "*k*-braid" (where *k* represents the number of threads). In particular, the averaged metric (2.1.23) is called "2-braid" as it is "coiled" from 2 lines $ds_a{}^{(-)}$ and $ds_b{}^{(-)}$ (see Definition № 1.22.1).

Analogously, averaging metrics (2.1.17) and (2.1.18) leads to a "2-antibraid”.

$$ds_{ab}^{(+)2} = \tfrac{1}{2}(ds_a^{(+)2} + ds_b^{(+)2}) = -c^2dt^2 + \frac{r^2}{r^2 - r_0^2}dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\varphi^2, \quad (2.1.36)$$

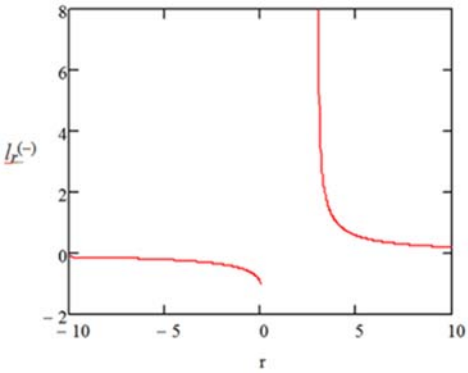
which describes the metric-dynamic state of the stable formation of a vacuum-type "air bubble in a liquid", but is a complete antithesis of the vacuum formation, describing a 2-braid (2.1.23). In such a case it behooves us to emphasize that the distance between two points r_1 and r_2 in the region with signature $(- + + +)$ is determined by the expression

$$r_2 - r_1 = \int_{r_1}^{r_2} \sqrt{g_{11}^{(+)}}\,dr\ .$$

The 2-braid (2.1.23) and 2-antibraid (1.36) fully complement one another, thereby yielding a solution of (2.1.21).

In total, the 2-braid (2.1.23) and 2-antibraid (2.1.36) completely compensate for each other's manifestations and give a solution (2.1.21): $ds_{ab}{}^{(-)2} + ds_{ab}{}^{(+)2} = ds^{(0)2}$. If it is conditionally assumed that the 2-braid (2.1.23) describes the metric-dynamic state of a stable “convexity” in the vacuum extent (Figures 2.1.1 and 2.1.2), then the 2-antibraid (2.1.36) describes exactly the same “concavity” in the same extent.

Substituting the components $g_{ii}^{0(-)}$ of metric (2.1.16) and component $g_{11}^{(-)}$ of the metrics (2.1.14) or (2.1.15) into equation (2.1.32) leads to the absurd results shown in Figure 2.1.5.



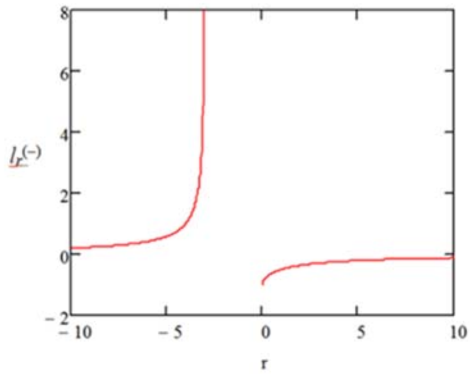


Fig. 2.1.5. a) Graph of the function $l_r^{(-)} = \sqrt{\dfrac{r}{r - r_s}} - 1$; b) Graph of the function $l_r^{(-)} = \sqrt{\dfrac{r}{r + r_s}} - 1$

The absurdity of the calculation results shown in Figure 2.1.5, once again confirms that the averaging of metrics (2.1.14) & (2.1.15) and/or metrics (2.1.17) & (2.1.18) is not meaningless, because this averaging leads to comprehensible results (see Figures 2.1.1 and 2.1.2).

Now we discuss the metric-dynamic interpretation of the zero components $g_{00}^{(-)}$ and $g_{00}^{(+)}$ of the metric tensors.

We introduce the usage of the terms "external" and "internal" (and related terms: outer, outside, inside, internal, etc.) to describe the same vacuum region by two metrics with mutually opposite signatures. The lengths in the local "external" and "internal" vacuum regions are given by pseudo-Euclidean metrics (2.1.16) and (2.1.19) (*see § 1.21*)

$$ds^{(-)2} = c^2dt^2 - dr^2 - r^2d\theta^2 - r^2\sin^2\theta\, d\varphi^2 = cdt'cdt'' - dx'dx'' - dy'dy'' - dz'dz'', \quad (2.1.37)$$

$$ds^{(+)2} = -c^2dt^2 + dr^2 + r^2d\theta^2 + r^2\sin^2\theta\, d\varphi^2 = -cdt'cdt'' + dx'dx'' + dy'dy'' + dz'dz''. \quad (2.1.38)$$

We introduce the terms for linear (affine) forms:

$ds^{(-)\prime} = cdt' - dx' - dy' - dz'$ – "Cover" on the outer side of the vacuum; (2.1.39)

$ds^{(-)\prime\prime} = cdt'' - dx'' - dy'' - dz''$ – "Inversion" of the outer side of the vacuum; (2.1.40)

$ds^{(+)\prime} = -cdt' + dx' + dy' + dz'$ – "Cover" of the inner side of the vacuum; (2.1.41)

$ds^{(+)\prime\prime} = -cdt'' + dx'' + dy'' + dz''$ – "Inversion" of the inner side of the vacuum. (2.1.42)

Let the "cover" and "inversion" of one side of the vacuum move relative to their initially fixed state along an axis x with the same velocity v_x, but in different directions. This is formally expressed by the coordinate transformation:

$$t' = t, \quad x' = x + v_x t, \quad y' = y, \quad z' = z \quad \text{– For the "cover",} \tag{2.1.43}$$

$$t'' = t, \quad x'' = x - v_x t, \quad y'' = y, \quad z'' = z \quad \text{– For the "inversion".} \tag{2.1.44}$$

A consequence of the equality of the velocities v_x in the modules of the "covers" and "inversions" due to the vacuum condition is that for every movement in the vacuum region there is a corresponding contrary movement [22].

Differentiating (2.1.43) and (2.1.44) and substituting the results of the differentiation to (2.1.37) and (1.38) in spherical coordinates we obtain metrics

$$ds_v^{(-)2} = (1 + v_r^{(-)2}/c^2)c^2dt^2 - dr^2 - r^2d\theta^2 - r^2\sin^2\theta\, d\varphi^2, \tag{2.1.45}$$

$$ds_v^{(+)2} = -(1 + v_r^{(+)2}/c^2)c^2dt^2 + dr^2 + r^2d\theta^2 + r^2\sin^2\theta\, d\varphi^2, \tag{2.1.46}$$

describing the kinematics of translational motion of the "external" and "internal" sides of the local area of the vacuum region. It then is under the vacuum conditions:

$$ds_v^{(-)2} + ds_v^{(+)2} = ds^{(0)2} = 0, \tag{2.1.47}$$

The movement involved is compensated for by the contrary movement.

Compare $g_{00}^{(-)}$ into the metrics (2.1.14) and (2.1.15) with $g_{00}^{(-)}$ in the metric (2.1.45) and $g_{00}^{(+)}$ in the metrics (2.1.17) and (2.1.18) with $g_{00}^{(+)}$ in the metric (2.1.46) respectively obtain:

for the metric (1.14): $1 - r_0/r = 1 + v_r^{(-a)2}/c^2 \;\rightarrow\; v_r^{(-a)2} = -c^2r_0/r \;\rightarrow\; v_r^{(-a)} = (-c^2r_0/r)^{½}$; (2.1.48)

for the metric (1.15): $1 + r_0/r = 1 + v_r^{(-b)2}/c^2 \;\rightarrow\; v_r^{(-b)2} = c^2r_0/r \;\rightarrow\; v_r^{(-b)} = (c^2r_0/r)^{½}$; (2.1.49)

for t e metric (1.17): $-(1 - r_0/r) = -(1 + v_r^{(+a)2}/c^2) \;\rightarrow\; v_r^{(+a)2} = -c^2r_0/r \;\rightarrow\; v_r^{(+a)} = (-c^2r_0/r)^{½}$; (2.1.50)

for the metric (1.18): $-(1 + r_0/r) = -(1 + v_r^{(+b)2}/c^2) \;\rightarrow\; v_r^{(+b)2} = c^2r_0/r \;\rightarrow\; v_r^{(+b)} = (c^2r_0/r)^{½}$ (2.1.51)

These results suggest that the zero components $g_{00}^{(-)}$ of the metrics (2.1.14) & (2.1.15) and $g_{00}^{(+)}$ of the metrics (2.1.17) & (2.1.18) describe the motion of the relevant sub-layer of the vacuum region with speeds v_r, as in (1.48) through (2.1.51), relative to their stationary state metrics given by (2.1.16) & (2.1.19).

Although we have movement, precisely what is moving in the vacuum state is not known, because there is no mechanism in the description of matter in geometrophysics to detect it. However, for convenience, in a vacuum such processes can be compared with processes in a elastoplastic fluids. **Terminology:** *We invented the terms "subcont", abbreviating "substantial continuum", and, correspondingly, "antisubcont" to designate the components of such an environment. Provisional names of the layers of vacuum region are given in Table 2.1.1.*

Table 2.1.1

<table>
<tr><th>Metric/ signature</th><th>Number of metric</th><th>Provisional name of terms</th><th>Side of vacuum</th></tr>
<tr><td>$ds_a^{(-)2}$
$(+---)$</td><td>(2.1.14)</td><td>"a-subcont" - the outer side of the outer side of the vacuum region</td><td rowspan="3">E X T E R N A L</td></tr>
<tr><td>$ds_b^{(-)2}$
$(+---)$</td><td>(2.1.15)</td><td>«b-subcont" - the inner side of the outer side of the vacuum region</td></tr>
<tr><td>$ds_c^{(-)2}$
$(+---)$</td><td>(2.1.16)</td><td>original flat outer side of the vacuum region</td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td>$ds_a^{(+)2}$
$(-+++)$</td><td>(2.1.17)</td><td>"a-antisubcont" - the outer side of the inner side of the vacuum region</td><td rowspan="3">I N T E R N A L</td></tr>
<tr><td>$ds_b^{(+)2}$
$(-+++)$</td><td>(2.1.18)</td><td>"b-antisubcont"- the inner side of the inner side of the vacuum region</td></tr>
<tr><td>$ds_c^{(+)2}$
$(-+++)$</td><td>(2.1.19)</td><td>original flat inner side of the vacuum region</td></tr>
</table>

Averaging the velocities (2.1.48) and (2.1.49), we find that the total motion of the affine layers of the outer side of the vacuum region (subcont) is described by the average velocity

$$v_{rab}^{(-)}(r) = ½[(-c^2r_0/r)^{½} + i(c^2r_0/r)^{½}], \quad (2.1.52)$$

and the velocity average (2.1.50) and (2.1.51), leads to the average velocity

$$v_{rab}^{(+)}(r) = ½[(-c^2r_0/r)^{½} + i(c^2r_0/r)^{½}]. \quad (2.1.53)$$

which describes the average (total) movement of the affine layer of the inside of the vacuum region (of the antisubcont).

The modules of the complex functions (2.1.52) and (2.1.53) are equal

$$|v_{rab}^{(-)}(r)| = 0, \quad (2.1.54)$$

$$|v_{rab}^{(+)}(r)| = 0, \quad (2.1.55)$$

which shows that the average velocity in the affine layers of the outer and inner sides of the vacuum region (subcont and antisubcont) with $r_0 = r$ is close to $\frac{\sqrt{2}}{2}c$, with c – the speed of light, but as the radius r increases greater than r_0, the velocity decreases in proportion to $1/r^{½}$, approaching zero.

However, the squares of the velocities (2.1.48) and (2.1.49) are equal and opposite to one other $v_{ra}^{(-)2} = -v_{rb}^{(-)2}$. Therefore, in the 2-braid (2.1.23), $g_{00}^{(-)} = 1$.

Similarly, the squared velocities (2.1.50) and (2.1.51) are equal and opposite each other $v_{ra}^{(-)2} = -v_{rb}^{(-)2}$. Therefore, in the 2-antibraid in (2.1.36), $g_{00}^{(+)} = 1$.

This circumstance determines the stability of the vacuum formation under consideration, since the number of "flowing" *a*-subcont is equal to the number of "flowing" *b*-subcont.

It should be noted that some additive combinations of metrics (2.1.14) through (2.1.16) and/or (2.1.17) through (2.1.19) are different solutions of the nonlinear Einstein field equations (2.1.6), leading to a more balanced metric-dynamic description of the local centrally symmetric vacuum formation than any one of them individually. The kinematics and dynamics of these vacuum layers and sub-layers are discussed in the following chapters.

2.2 The second vacuum second Einstein's vacuum equations and their solutions

Taking into account the following covariant derivatives of tensors are equal to zero:

$$\nabla_j g_{ik} = 0, \tag{2.2.1}$$

$$\nabla_j (R_{ik} - \frac{1}{2} R g_{ik}) = 0, \tag{2.2.2}$$

Einstein supplemented equation (2.1.1) with another term (the so-called Λ-term)

$$R_{ik} - \frac{1}{2} R g_{ik} + \Lambda g_{ik} = 0, \tag{2.2.3}$$

in the literature on general relativity often takes $\Lambda = \pm 3/r_a^2$ = constant, r_a is the radius of the spherical vacuum formation.

In this case

$$g^{ik}\left\{R_{ik} - \frac{1}{2} R g_{ik} + \Lambda g_{ik}\right\} = R - \frac{n}{2} R + n\Lambda = 0, \tag{2.2.4}$$

whence

$$R = \frac{2n}{n-2}\Lambda, \tag{2.2.5}$$

whereupon the equation (2.2.3) takes the form

$$R_{ik} - \frac{n}{n-2}\Lambda g_{ik} + \Lambda g_{ik} = R_{ik} - \frac{2}{n-2}\Lambda g_{ik} = 0. \tag{2.2.6}$$

For 4-dimensional space: $n = 4$, $R = 4\Lambda$, equation (2.2.6) takes the most simple form

$$R_{ik} - \Lambda g_{ik} = 0 \quad \text{or} \quad R_{ik} = \pm\frac{3}{r_a^2} g_{ik} = \begin{cases} R_{ik} = \dfrac{3}{r_a^2} g_{ik}, \\ R_{ik} = -\dfrac{3}{r_a^2} g_{ik}. \end{cases} \tag{2.2.7}$$

Equations (2.2.7) will be called the second Einstein's vacuum equations.

The solutions of the second vacuum equation (2.2.7) are the following set of generalized Kottler metrics with a signature (+ − − −), which we will arbitrarily call "convexity" in the vacuum extent:

$$ds_1^{(-)2} = \left(1 - \frac{r_b}{r} + \frac{r^2}{r_a^2}\right)c^2dt^2 - \frac{dr^2}{\left(1 - \frac{r_b}{r} + \frac{r^2}{r_a^2}\right)} - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \tag{2.2.8}$$

$$ds_2^{(-)2} = \left(1 + \frac{r_b}{r} - \frac{r^2}{r_a^2}\right)c^2dt^2 - \frac{dr^2}{\left(1 + \frac{r_b}{r} - \frac{r^2}{r_a^2}\right)} - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \tag{2.2.9}$$

$$ds_3^{(-)2} = \left(1 - \frac{r_b}{r} - \frac{r^2}{r_a^2}\right)c^2dt^2 - \frac{dr^2}{\left(1 - \frac{r_b}{r} - \frac{r^2}{r_a^2}\right)} - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \tag{2.2.10}$$

$$ds_4^{(-)2} = \left(1 + \frac{r_b}{r} + \frac{r^2}{r_a^2}\right)c^2dt^2 - \frac{dr^2}{\left(1 + \frac{r_b}{r} + \frac{r^2}{r_a^2}\right)} - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \tag{2.2.11}$$

$$ds_5^{(-)2} = c^2dt^2 - dr^2 - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right); \tag{2.2.12}$$

and a set of generalized Kottler metrics with a signature $(-\,+\,+\,+)$, which we will arbitrarily call "concavity" in the vacuum extent:

$$ds_1^{(+)2} = -\left(1 - \frac{r_b}{r} + \frac{r^2}{r_a^2}\right)c^2dt^2 + \frac{dr^2}{\left(1 - \frac{r_b}{r} + \frac{r^2}{r_a^2}\right)} + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \tag{2.2.13}$$

$$ds_2^{(+)2} = -\left(1 + \frac{r_b}{r} - \frac{r^2}{r_a^2}\right)c^2dt^2 + \frac{dr^2}{\left(1 + \frac{r_b}{r} - \frac{r^2}{r_a^2}\right)} + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \tag{2.2.14}$$

$$ds_3^{(+)2} = -\left(1 - \frac{r_b}{r} - \frac{r^2}{r_a^2}\right)c^2dt^2 + \frac{dr^2}{\left(1 - \frac{r_b}{r} - \frac{r^2}{r_a^2}\right)} + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \tag{2.2.15}$$

$$ds_4^{(+)2} = -\left(1 + \frac{r_b}{r} + \frac{r^2}{r_a^2}\right)c^2dt^2 + \frac{dr^2}{\left(1 + \frac{r_b}{r} + \frac{r^2}{r_a^2}\right)} + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \tag{2.2.16}$$

$$ds_5^{(+)2} = -\,c^2dt^2 + dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right). \tag{2.2.17}$$

where r_b is the constant of integration, analogous to $b = r_0$ in the solutions (2.1.13).

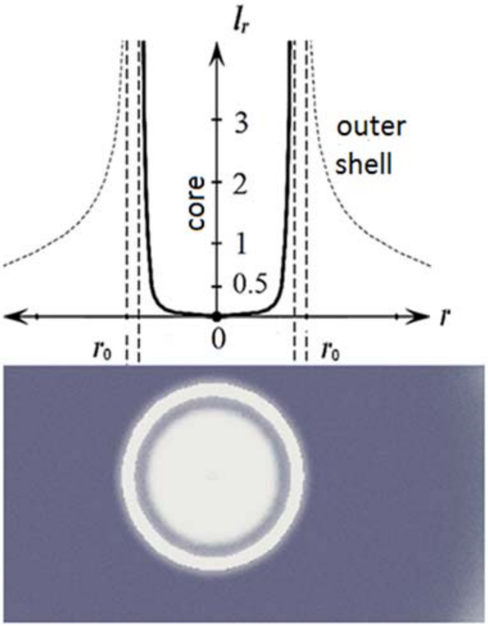


Fig. 2.2.1 Graph of $l_{rc}^{(-)}$ function - elongation of the vacuum extension in the core (i.e. within a spherical cavity)

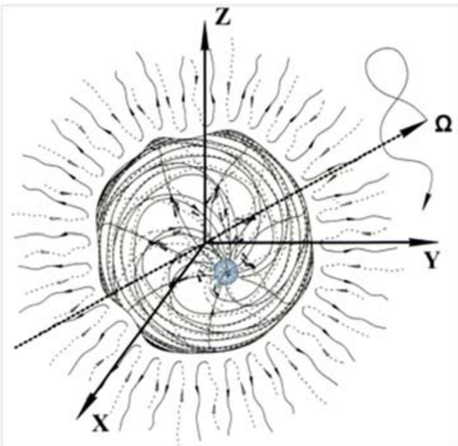


Fig. 2.2.2 Rotation of the core of the vacuum formation

Wherein, the metrics (2.2.12) and (2.2.17) are particular cases of the metrics (2.2.8) – (2.2.11) and (2.2.13) – (2.2.16), respectively, using $r_b = 0$ and $r_a = \infty$.

The sum of all metrics (2.2.8) through (2.2.17) again leads to the metric (2.1.21), which is also a trivial solution of (2.2.7).

When $r_a = \infty$ and $r_b \neq 0$, the generalized Kottler metrics (2.2.8) through (2.2.17) is transformed into the generalized Schwarzschild metrics (2.1.14) through (2.1.19), while for $r_b = 0$ and $1/r_a = 1/r_0 \neq 0$, the metrics (2.2.8) through (2.2.17) become the de Sitter metrics:

- for the convex vacuum region (bulge), with signature (+ – – –):

$$ds_a^{(-)2} = \left(1+\frac{r^2}{r_0^2}\right)c^2dt^2 - \frac{dr^2}{\left(1+\frac{r^2}{r_0^2}\right)} - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \tag{2.2.18}$$

$$ds_b^{(-)2} = \left(1-\frac{r^2}{r_0^2}\right)c^2dt^2 - \frac{dr^2}{\left(1-\frac{r^2}{r_0^2}\right)} - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \tag{2.2.19}$$

$$ds_c^{(-)2} = c^2dt^2 - dr^2 - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right); \tag{2.2.20}$$

- for the concave vacuum region (concavity) with signature (– + + +):

$$ds_a^{(+)2} = -\left(1+\frac{r^2}{r_0^2}\right)c^2dt^2 + \frac{dr^2}{\left(1+\frac{r^2}{r_0^2}\right)} + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \tag{2.2.21}$$

$$ds_b^{(+)2} = -\left(1-\frac{r^2}{r_0^2}\right)c^2dt^2 + \frac{dr^2}{\left(1-\frac{r^2}{r_0^2}\right)} + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \tag{2.2.22}$$

$$ds_c^{(+)2} = -\,c^2dt^2 + dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right). \tag{2.2.23}$$

When $r_a = r_0$, the metrics (2.2.18) and (2.2.19) describes a closed convex (spherical) vacuum formation (i.e., the "core") in the range [0, r_0] (Figure 2.2.1 and 2.2.2). This describes a region that has been defined as a vacuum cavity in the solution of the first vacuum equations (2.1.6) (Figure 2.1.2).

The arithmetic average of the two metrics (2.2.18) and (2.2.19) forms a 2-braid:

$$ds_{ab}^{(-)2} = c^2dt^2 - \frac{dr^2}{\left(1-\frac{r^4}{r_0^4}\right)} - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right). \tag{2.2.24}$$

Substituting components $g_{ii}^{0(-)}$ and $g_{ii}^{(-)}$, respectively, from (2.2.20) and (2.2.24) into (2.1.32), we obtain the relative lengthening of the one of side of vacuum

$$l_t^{(-)} = 0, \quad l_{rc}^{(-)} = \frac{\Delta r}{r} - 1 = \sqrt{\frac{r_0^4}{r_0^4 - r^4}} - 1, \quad l_\theta^{(-)} = 0, \quad l_\varphi^{(-)} = 0. \tag{2.2.25}$$

The graph of the function $l_{rc}^{(-)}$ (2.2.25) (relative elongation of the vacuum extension in the radial direction in the core) is shown in Figure 2.2.1.

Thus, Einstein's second vacuum equation (2.2.7) describes not only the outer shell of the vacuum formation surrounding the spherical cavity (Figure 2.1.2), but also the core of this vacuum formation filling this cavity (Figure 2.2.1, 2.2.2).

In the general case, metrics (2.8) through (2.11) should be written as:

$$ds_1^{(-)2} = \left(1 - \frac{r_{b1}}{r} + \frac{r^2}{r_{a1}^2}\right)c^2dt^2 - \frac{dr^2}{\left(1 - \frac{r_{b1}}{r} + \frac{r^2}{r_{a1}^2}\right)} - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \tag{2.2.25a}$$

$$ds_2^{(-)2} = \left(1 + \frac{r_{b2}}{r} - \frac{r^2}{r_{a2}^2}\right)c^2dt^2 - \frac{dr^2}{\left(1 + \frac{r_{b2}}{r} - \frac{r^2}{r_{a2}^2}\right)} - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right),$$

$$ds_3^{(-)2} = \left(1 - \frac{r_{b3}}{r} - \frac{r^2}{r_{a3}^2}\right)c^2dt^2 - \frac{dr^2}{\left(1 - \frac{r_{b3}}{r} - \frac{r^2}{r_{a3}^2}\right)} - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right),$$

$$ds_4^{(-)2} = \left(1 + \frac{r_{b4}}{r} + \frac{r^2}{r_{a4}^2}\right)c^2dt^2 - \frac{dr^2}{\left(1 + \frac{r_{b4}}{r} + \frac{r^2}{r_{a4}^2}\right)} - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right).$$

In this case, the 4-braid composed of metrics (2.2.25a)

$$ds_{1\text{-}4}^{(-)2} = \tfrac{1}{4}\left(ds_1^{(-)2} + ds_2^{(-)2} + ds_3^{(-)2} + ds_4^{(-)2}\right), \tag{2.2.25b}$$

can be recorded as

$$ds_{1-4}^{(-)2} = f(r)c^2dt^2 - k(r)dr^2 - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right),$$

where

$$f(r) = \frac{1}{4}\left[\left(1-\frac{r_{b1}}{r}+\frac{r^2}{r_{a1}^2}\right)+\left(1-\frac{r_{b2}}{r}+\frac{r^2}{r_{a2}^2}\right)+\left(1-\frac{r_{b3}}{r}+\frac{r^2}{r_{a3}^2}\right)+\left(1-\frac{r_{b4}}{r}+\frac{r^2}{r_{a4}^2}\right)\right]$$

$$k(r) = \frac{1}{4}\left[\frac{1}{\left(1-\frac{r_{b1}}{r}+\frac{r^2}{r_{a1}^2}\right)}+\frac{1}{\left(1-\frac{r_{b2}}{r}+\frac{r^2}{r_{a2}^2}\right)}+\frac{1}{\left(1-\frac{r_{b3}}{r}+\frac{r^2}{r_{a3}^2}\right)}+\frac{1}{\left(1-\frac{r_{b4}}{r}+\frac{r^2}{r_{a4}^2}\right)}\right]$$

Such a 4-braid $ds_{1\text{-}4}^{(-)2}$ is formed by eight twisted “threads” (i.e. linear forms) $ds_i^{(-)}$ forming a system of two complex conjugated quaternions:

$$ds_{1\text{-}4}^{(-)\prime} = \tfrac{1}{\sqrt{4}}\,(ds_1^{(-)\prime} + \mathbf{i}ds_2^{(-)\prime} + \mathbf{j}ds_3^{(-)\prime} + \mathbf{k}ds_4^{(-)\prime}), \qquad (2.2.26)$$

$$ds_{1\text{-}4}^{(-)\prime\prime} = \tfrac{1}{\sqrt{4}}\,(ds_1^{(-)\prime\prime} - \mathbf{i}ds_2^{(-)\prime\prime} - \mathbf{j}ds_3^{(-)\prime\prime} - \mathbf{k}ds_4^{(-)\prime\prime}),$$

whose product is equal to (2.2.25b).   .

Comparing $g_{00}^{(-)}$ in the metrics (2.2.18) and (2.2.19) with $g_{00}^{(-)}$ in the metric (2.1.45) and $g_{00}^{(+)}$ in the metrics (2.2.21) and (2.2.22) and with $g_{00}^{(+)}$ in the metric (2.1.46), we find the speed of movement of the vacuum layers at each point of the "core" of the vacuum formation (Figure 2.2.1):

for the metric (2.2.18): $1 + r^2/r_0^2 = 1 + v_{ra}^{(-)2}/c^2 \rightarrow v_{ra}^{(-)2} = c^2r^2/r_0^2 \rightarrow v_{ra}^{(-)} = cr/r_0$; (2.2.27)

for the metric (2.2.19): $1 - r^2/r_0^2 = 1 + v_{rb}^{(-)2}/c^2 \rightarrow v_{rb}^{(-)2} = -c^2r^2/r_0^2 \rightarrow v_{rb}^{(-)} = -\,cr/r_0$; (2.2.28)

for the metric (2.2.21): $-(1 + r^2/r_0^2) = -(1 + v_{ra}^{(+)2}/c^2) \rightarrow v_{ra}^{(+)2} = c^2r^2/r_0^2 \rightarrow v_{ra}^{(+)} = cr/r_0$; (2.2.29)

for the metric (2.2.22): $-(1 - r^2/r_0^2) = -(1 + v_{rb}^{(+)2}/c^2) \rightarrow v_{rb}^{(+)2} = -c^2r^2/r_0^2 \rightarrow v_{rb}^{(+)} = -\,cr/r_0$. (2.2.30)

From the expression (2.2.27) through (2.2.28) of the movements in mutually opposite directions, we find that the speed of the vacuum layers $v_{ra}^{(-)} = -\,v_{rb}^{(-)}$ in the center of the core (at $r = 0$; Figure 2.2.1) is zero, and on the periphery of the core with radius r_0, they move with the speed of light c.

More physical is the situation when the core of the vacuum formation rotates. According to the classification given in the Table 2.1.1, the a-subcont rotates in the periphery of the core at the speed of light $v_{ra}^{(-)}(r_0) = c$ (Figure 2.2.2). Then the a-subcont spirals inward, decelerating as it approaches the center of the core, where it stops [$v_{ra}^{(-)}(0) = 0$] and turns into a b-subcont. In its turn, the *b*-subcont flows outward in a spiral from the center of the core, starting with the velocity $v_{rb}^{(-)}(0) = 0$ and accelerating, ending with its rotation at the periphery of the core at the speed of light ($v_{rb}^{(-)}(r_0) = c$) (Figure 2.2.2), where it is converted into an *a*-subcont.

Thus, the intra-core *ab*-subcont "processes" the loop, and support the strongly deformed periphery of the core of the vacuum formation (Figure 2.2.1) at a steady state.

2.3 The non-Riemannian geometry with torsion and rotation

In the previous paragraph, it was noted that the study of stable vacuum entities should take into account the rotation of their "core", therefore touch on some aspects of geometry with torsion and rotation.

Of all non-Riemannian geometries, one of the most important is the geometry of Riemann-Cartan space with absolute parallelism, which was often used by Einstein [50, 54]. The Riemann-Christoffel curvature tensor uses this, as given in [27]. We the curvature equal to zero as follows:

$$R^{\beta}_{\beta\mu\nu}(Q) = R^{\beta}_{\beta\mu\nu} + K^{\alpha}_{\beta\nu;\mu} - K^{\alpha}_{\beta\mu;\nu} + K^{\alpha}_{\mu\sigma}K^{\sigma}_{\beta\nu} - K^{\alpha}_{\nu\sigma}K^{\sigma}_{\beta\mu} = 0, \qquad (2.3.1)$$

where the Riemann curvature tensor is $R^{\beta}_{\beta\mu\nu} = \frac{\delta}{\delta x^{\mu}}\Gamma^{\beta}_{\beta\nu} - \frac{\delta}{\delta x^{\nu}}\Gamma^{\beta}_{\beta\mu} + \Gamma^{\beta}_{\mu\sigma}\Gamma^{\sigma}_{\beta\nu} - \Gamma^{\beta}_{\nu\sigma}\Gamma^{\sigma}_{\beta\mu}$ and the other terms are based on the contortion tensor, using the lowering of the indices via $Q_{\mu\nu\lambda} = g_{\lambda\alpha}Q^{\alpha}_{\mu\nu}$

$$K_{\mu\nu\lambda} = \frac{1}{2}\left(Q_{\mu\nu\lambda} - Q_{\nu\lambda\mu} + Q_{\lambda\mu\nu}\right), \qquad (2.3.2)$$

which in its turn is based (by lowering of the indices) on the torsion tensor

$$Q^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu} - \Gamma^{\lambda}_{\nu\mu}. \qquad (2.3.3)$$

The identity (2.3.1) means that in absolute parallelism geometry, the components of the Riemann curvature tensor are fully compensated by torsion. In this case, instead of (2.2.7) in the geometry based on the variational principle, one obtains the Einstein-Cartan equation [27]

$$R_{\mu\nu} - \tfrac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = Y_{\mu\nu}, \qquad (2.3.4)$$

where $$Y_{\mu\nu} = K_{\mu}K_{\nu} + K_{\mu\alpha\beta}K^{\alpha\beta}_{\nu} + K_{\alpha\mu\beta}K^{\beta\alpha}_{\nu} + K_{\alpha\beta\mu}K^{\alpha\beta}_{\nu} - \tfrac{1}{2}g_{\mu\nu}\left(K_{\lambda}K^{\lambda} + K_{\lambda\mu\nu}K^{\lambda\mu\nu}\right) \qquad (2.3.5)$$

$K_{\nu} = 2Q_{\nu} = Q^{\lambda}_{\nu\lambda}$ is the trace of the contortion tensor. (2.3.6)

This equation looks as if the torsion of space, or rather the rotational inertia as explained in [49], is the source of its curvature. Investigating more closely, one sees that the converse is the case, whereby the curvature of space is the source of its torsion.

In the works of R. Vaytsenbeka, D. Vitali and G. Shipov, absolute parallelism geometry also received full geometrized treatment using the equation [49]

$$R_{jm} - \tfrac{1}{2}Rg_{jm} + \Lambda g_{jm} = -\Phi_{jm}, \qquad (2.3.7)$$

where the right side is expressed in the formal terms of reference:

$$\Phi_{jm} = 2\left\{\nabla_{[i}\Phi^{i}_{|j|m]} + \Phi^{i}_{s[i}\Phi^{s}_{|j|m]} - \frac{1}{2}g_{jm}g^{pn}\left(\nabla_{[i}\Phi^{i}_{|p|n]} + \Phi^{i}_{s[i}\Phi^{s}_{|p|n]}\right)\right\}; \qquad (2.3.8)$$

is the Vaytsenbek - Vitali - Shipov tensor;

$$\Phi^{i}_{jk} = -\Omega^{..i}_{jk} + g^{im}\left(g_{is}\Omega^{..s}_{mk} + g_{ks}\Omega^{..s}_{mj}\right) \tag{2.3.9}$$

is the Ricci rotation coefficients;

$$\Omega^{i}_{jk} = \tfrac{1}{2} e^{i}_{a}\left(e^{a}_{k,j} - e^{a}_{j,k}\right) \tag{2.3.10}$$

is the non-holonomicity object; e_{ak} are components of the unit vector of a rotating 3-D reference basis.

Different approaches by Cartan - Schouten and Vaytsenbek - Vitali - Shipov to constructing geometries with torsion and rotation characterize the different types of rotational space. If the $Y_{\mu\nu}$ tensor characterizes the motion vector at the start of the trial and the curved region of the rotating vacuum, the tensor Φ_{ik} characterizes the torsional rotation around the center of reference in 3 dimensions.

In general, the equation is fully geometrized

$$R_{\mu\nu} - \tfrac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = Y_{\mu\nu} + \Phi_{\mu\nu}. \tag{2.3.11}$$

However, existence not equal to zero of the right-hand sides of equations (2.3.6), (2.3.7) and (2.3.11) leads inevitably to an unstable condition of the vacuum region, because tensors $Y_{\mu\nu}$ and $F_{\mu\nu}$ are both nonzero, so that they obey:

$$\nabla_j (Y_{ik} + \Phi_{ik}) = \frac{\partial (Y_{ik} + \Phi_{ik})}{\partial x^j} - \Gamma^{l}_{ij}(Y_{ik} + \Phi_{ik}) - \Gamma^{l}_{kj}(Y_{ik} + \Phi_{ik}) = 0, \tag{2.3.12}$$

instead of the law of conservation

$$\partial (Y_{ik} + \Phi_{ik})/\partial x^k = 0, \tag{2.3.13}$$

Thus, for stable vacuum formations all the components of the Cartan-Schouten tensor $Y_{\mu\nu}$ and the Vaytsenbek - Vitali - Shipov tensor Φ_{ik} should be equal to zero. Thus the identity (2.3.11) falls into a system of two or three equations

$$\begin{cases} R_{\mu\nu} - \tfrac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 0, \\ Y_{\mu\nu} + \Phi_{\mu\nu} = 0, \end{cases} \qquad \begin{cases} R_{\mu\nu} - \tfrac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 0, \\ Y_{\mu\nu} = 0, \\ \Phi_{\mu\nu} = 0. \end{cases} \tag{2.3.14}$$

It is important to note that in Riemann - Cartan, space is unbalanced due to the asymmetry of the Christoffel symbol and the Ricci tensor $R_{\mu\nu} \neq R_{\nu\mu}$. But in the particular case of $\Lambda = 0$, $Y_{\mu\nu} = 0$ and $\Phi_{\mu\nu} = 0$ (or $Y_{\mu\nu} + \Phi_{\mu\nu} = 0$) of the equations (2.2.5) and (2.3.11), $R_{\mu\nu} = 0$ and $R_{\nu\mu} = 0$, so they are identically equal $R_{\mu\nu} \equiv R_{\nu\mu}$.

This corresponds to these types of spins and vacuum twists which do not affect the Ricci tensor $R_{\mu\nu}$, but they can influence the curvature tensor components. It seems that a certain amount of space is rotated with respect to the external observer, but those who are within its scope almost do not feel this rotation. As a rough example, it is very difficult to feel that the Earth's surface rotates for those on it. However, there are effects indicating the presence of inertial forces caused by the rotational motion of the planet, for example, the deviation of the pendulum of Foucault, the different slopes of the left and right banks of rivers, etc. It is this type of rotation of the core of the vacuum formation that we have treated in § 2.2 (Figure 2.2.2).

2.4 The extended (third) Einstein's vacuum equation

Up to this point, sets of solutions of Einstein's vacuum equations (2.1.6) and (2.2.7), well known to specialists, were considered. In this paragraph, it is proposed for the first time to consider an extended version of these equations.

Due to the properties of the components of the metric tensor (2.2.1), it is easy to show that

$$\nabla_j \Lambda g_{ik} = \Lambda \nabla_j g_{ik} = 0. \tag{2.4.1}$$

The following equality is also obvious

$$\nabla_j \left(\Lambda_1 g_{ik} + \Lambda_2 g_{ik} + \Lambda_3 g_{ik} + \ldots + \Lambda_\infty g_{ik}\right) = \Lambda_1 \nabla_j g_{ik} + \Lambda_2 \nabla_j g_{ik} + \Lambda_3 \nabla_j g_{ik} + \ldots + \Lambda_\infty \nabla_j g_{ik} = 0, \tag{2.4.2}$$

where $\Lambda_1, \Lambda_2, \ldots, \Lambda_\infty$ are constants.

Therefore, guided by the same considerations that led Einstein to introduce Λ as a member of equation (2.2.3), we can write

$$R_{ik} - \frac{1}{2} R g_{ik} + \Lambda_1 g_{ik} + \Lambda_2 g_{ik} + \Lambda_3 g_{ik} + \ldots + \Lambda_\infty g_{ik} = 0, \tag{2.4.3}$$

or

$$R_{ik} - \frac{1}{2} R g_{ik} + \left(\Lambda_1 + \Lambda_2 + \Lambda_3 + \ldots + \Lambda_\infty\right) g_{ik} = 0, \tag{2.4.4}$$

where $\Lambda_k = \pm 3/r_k^2$, here r_k is the radius of the k^{th} spherical vacuum formation.

If $\Lambda_1 + \Lambda_2 + \Lambda_3 + \ldots + \Lambda_\infty = \Lambda_0$ (i.e., if the sum of this series converges to Λ_0), then equation (2.4.4) is reduced to the form of equation (2.2.3).

Indeed, in this case, equation (4.4) reduces to the form (2.2.3)

$$R_{ik} - \frac{1}{2} R g_{ik} + \Lambda_0 g_{ik} = 0. \tag{2.4.5}$$

Combining equation (2.4.4) with g_{ik}, we get

$$g^{ik}\left\{R_{ik}-\frac{1}{2}Rg_{ik}+\left(\Lambda_1+\Lambda_2+\Lambda_3+...+\Lambda_\infty\right)g_{ik}\right\}=R-\frac{n}{2}R+n\left(\Lambda_1+\Lambda_2+\Lambda_3+...+\Lambda_\infty\right)=0, \quad (2.4.6)$$

whence

$$R=\frac{2n}{n-2}\sum_{k=1}^{\infty}\Lambda_k=\frac{2n}{n-2}\Lambda_0. \quad (2.4.7)$$

Substituting (2.4.7) into (2.4.6), for $n = 4$ we obtain the simplest (for the case) version of the extended Einstein's vacuum equation

$$R_{ik}-g_{ik}\sum_{k=1}^{\infty}\Lambda_k=0. \quad (2.4.8)$$

This expression will be called the "third Einstein's vacuum equation”

The series in equation (2.4.4), taking into account (2.4.7) and $n = 4$, converges to $R/4$ either:

absolutely: $$\Lambda_0=\sum_{k=1}^{\infty}\Lambda_k=3\sum_{k=1}^{\infty}\frac{N_k}{r_k^2}=\frac{R}{4}, \quad (2.4.9)$$

or

sign-variable: $$\Lambda_0=3\sum_{k=1}^{\infty}\left(-1\right)^k\frac{N_k}{r_k^2}=\frac{R}{4}. \quad (2.4.10)$$

where N_k represents a sequence of numbers.

Of particular interest is the average of the Ricci-flat vacuum region with $R_{ik} = 0$ because of its correlation with Ricci-flat Calabi-Yau spaces. In this case, according to (2.4.7) and (2.4.8), we have:

$$\Lambda_0=\sum_{k=1}^{\infty}\Lambda_k=0 \text{ and } R=0, \quad (2.4.11)$$

the system of equations (2.3.14) breaks up into a system of two or three equations:

$$\begin{cases} R_{\mu\nu}=0, \\ Y_{\mu\nu}+\Phi_{\mu\nu}=0, \\ \sum_{k=1}^{\infty}\Lambda_k=0 \end{cases} \qquad \begin{cases} R_{\mu\nu}=0, \\ Y_{\mu\nu}=0, \\ \Phi_{\mu\nu}=0, \\ \sum_{k=1}^{\infty}\Lambda_k=0 \end{cases} \quad (2.4.12)$$

2.5 Solutions of the third Einstein's vacuum equations

Consider the most important (in the opinion of the author) case when the third Einstein's vacuum equation (2.4.8) has the form

$$R_{ik} - g_{ik}\Lambda_0 = 0, \tag{2.5.1}$$

where
$$\Lambda_0 = \sum_{k=1}^{\infty}\Lambda_k = 3\sum_{k=1}^{\infty}(-1)^k\frac{N_k}{r_k^2} = 0 \tag{2.5.2}$$

is an alternating series which is equal to zero.

First of all, we find the solution of equation (2.5.1) for

$$\sum_{k=1}^{\infty}\Lambda_k = \Lambda_0 . \tag{2.5.3}$$

The form of the equation (2.5.1) completely coincides with the form of the second Einstein's vacuum equation (2.2.7). Therefore, the solutions of equation (2.5.1) are the generalized Kottler metrics similar to metrics (2.2.8) – (2.2.17):

- with signature (+ – – –), for a "convex" vacuum formation

$$ds_1^{(-)2} = \left(1 - \frac{r_f}{r} + \frac{\Lambda_0 r^2}{3}\right)c^2dt^2 - \frac{dr^2}{\left(1 - \frac{r_f}{r} + \frac{\Lambda_0 r^2}{3}\right)} - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \tag{2.5.4}$$

$$ds_2^{(-)2} = \left(1 + \frac{r_f}{r} - \frac{\Lambda_0 r^2}{3}\right)c^2dt^2 - \frac{dr^2}{\left(1 + \frac{r_f}{r} - \frac{\Lambda_0 r^2}{3}\right)} - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \tag{2.5.5}$$

$$ds_3^{(-)2} = \left(1 - \frac{r_f}{r} - \frac{\Lambda_0 r^2}{3}\right)c^2dt^2 - \frac{dr^2}{\left(1 - \frac{r_f}{r} - \frac{\Lambda_0 r^2}{3}\right)} - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \tag{2.5.6}$$

$$ds_4^{(-)2} = \left(1 + \frac{r_f}{r} + \frac{\Lambda_0 r^2}{3}\right)c^2dt^2 - \frac{dr^2}{\left(1 + \frac{r_f}{r} + \frac{\Lambda_0 r^2}{3}\right)} - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \tag{2.5.7}$$

$$ds_5^{(-)2} = c^2dt^2 - dr^2 - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right); \tag{2.5.8}$$

- with signature $(+ - - -)$, for a "convex" vacuum formation

$$ds_5^{(+)2} = -c^2dt^2 + dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \tag{2.5.9}$$

$$ds_4^{(+)2} = -\left(1+\frac{r_f}{r}+\frac{\Lambda_0 r^2}{3}\right)c^2dt^2 + \frac{dr^2}{\left(1+\frac{r_f}{r}+\frac{\Lambda_0 r^2}{3}\right)} + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \tag{2.5.10}$$

$$ds_3^{(+)2} = -\left(1-\frac{r_f}{r}-\frac{\Lambda_0 r^2}{3}\right)c^2dt^2 + \frac{dr^2}{\left(1-\frac{r_f}{r}-\frac{\Lambda_0 r^2}{3}\right)} + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \tag{2.5.11}$$

$$ds_2^{(+)2} = -\left(1+\frac{r_f}{r}-\frac{\Lambda_0 r^2}{3}\right)c^2dt^2 + \frac{dr^2}{\left(1+\frac{r_f}{r}-\frac{\Lambda_0 r^2}{3}\right)} + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \tag{2.5.12}$$

$$ds_1^{(+)2} = -\left(1-\frac{r_f}{r}+\frac{\Lambda_0 r^2}{3}\right)c^2dt^2 + \frac{dr^2}{\left(1-\frac{r_f}{r}+\frac{\Lambda_0 r^2}{3}\right)} + r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \tag{2.5.13}$$

where

$$\Lambda_0 = \sum_{k=1}^{\infty}\Lambda_k = \sum_{k=1}^{\infty}\frac{3N_k}{r_k^2} + \sum_{k=1}^{\infty} -\frac{3N_k}{r_k^2} = 0\ , \tag{2.5.14}$$

$$r_f = \sum_{k=1}^{\infty} r_k + \sum_{k=1}^{\infty} -r_k = 0, \tag{2.5.15}$$

whereby we may substitute $b = r_f$ in the solution to (2.1.13).

Further will be considered two private, but, in the author's opinion, important cases which we will conditionally call "Hierarchy of ten spheres" and "Lucas-Fibonacci Branches".

Terminology (translator's notes)**:** *The two cases may appear isolated, but together their solutions relate to one another in ways yielding unexpectedly fruitful results. The author considers these important enough to baptize them with names. Just as Gell-Mann could allude to a term from Buddhism to coin his Eightfold Way, so too we allude to a couple of terms out of an ancient Jewish tradition in order to coin our terms for the organization presented in the next two sections. The first set of results (§ 2.6) is dubbed the "Hierarchy of ten spheres", while the second (§ 2.7) is baptized "Lucas-Fibonacci Branches".*

2.6 Hierarchy of ten spheres

We investigate the special case where the series (2.5.14) and (2.5.15) have the simple form:

$$\Lambda_0 = \sum_{k=1}^{10} \Lambda_k = \sum_{k=1}^{10} \frac{3}{r_k^2} + \sum_{k=1}^{10} \left(-\frac{3}{r_k^2} \right) = 0, \tag{2.6.1}$$

$$r_f = \sum_{k=1}^{10} r_k + \sum_{k=1}^{10} (-r_k) = 0. \tag{2.6.2}$$

Consider a series of separate positive and negative terms

$$r_d = \sum_{k=1}^{10} r_k , \qquad \Lambda_d = 3\sum_{k=1}^{10} \frac{1}{r_k^2}, \tag{2.6.3}$$

$$r_{-d} = \sum_{k=1}^{10} - r_k , \qquad \Lambda_{-d} = 3\sum_{k=1}^{10} - \frac{1}{r_k^2}. \tag{2.6.4}$$

We substitute the series (2.6.3) in the metric (2.5.4) through (2.5.7) instead of the series (2.5.14) and (2.5.15) and take into account that we can write

$$\begin{aligned} 1-\frac{r_d}{r}+\frac{\Lambda_d r^2}{3} &= 1-\frac{r_1+r_2+...+r_{10}}{r}+\left(\frac{1}{r_1^2}+\frac{1}{r_2^2}+...+\frac{1}{r_{10}^2}\right)r^2 = \\ &= \left(1-\frac{r_{10}}{r}+\frac{r^2}{r_9^2}\right)-\left(1+\frac{r_9}{r}-\frac{r^2}{r_8^2}\right)+\left(1-\frac{r_8}{r}+\frac{r^2}{r_7^2}\right)-\left(1+\frac{r_7}{r}-\frac{r^2}{r_6^2}\right)+\left(1-\frac{r_6}{r}+\frac{r^2}{r_5^2}\right)-\left(1+\frac{r_5}{r}-\frac{r^2}{r_4^2}\right)+ \\ &+\left(1-\frac{r_4}{r}+\frac{r^2}{r_3^2}\right)-\left(1+\frac{r_3}{r}-\frac{r^2}{r_2^2}\right)+\left(1-\frac{r_2}{r}+\frac{r^2}{r_1^2}\right)-\left(1+\frac{r_1}{r}-\frac{r^2}{r_{10}^2}\right), \end{aligned} \tag{2.6.5}$$

$$\begin{aligned} 1+\frac{r_d}{r}-\frac{\Lambda_d r^2}{3} &= 1+\frac{r_1+r_2+...+r_{10}}{r}-\left(\frac{1}{r_1^2}+\frac{1}{r_2^2}+...+\frac{1}{r_{10}^2}\right)r^2 = \\ &= \left(1+\frac{r_{10}}{r}-\frac{r^2}{r_9^2}\right)-\left(1-\frac{r_9}{r}+\frac{r^2}{r_8^2}\right)+\left(1+\frac{r_8}{r}-\frac{r^2}{r_7^2}\right)-...+\left(1+\frac{r_2}{r}-\frac{r^2}{r_1^2}\right)-\left(1-\frac{r_1}{r}+\frac{r^2}{r_{10}^2}\right), \end{aligned} \tag{2.6.6}$$

$$\begin{aligned} 1-\frac{r_d}{r}-\frac{\Lambda_d r^2}{3} &= 1-\frac{r_1+r_2+...+r_{10}}{r}-\left(\frac{1}{r_1^2}+\frac{1}{r_2^2}+...+\frac{1}{r_{10}^2}\right)r^2 = \\ &= \left(1-\frac{r_{10}}{r}-\frac{r^2}{r_9^2}\right)-\left(1+\frac{r_9}{r}+\frac{r^2}{r_8^2}\right)+\left(1-\frac{r_8}{r}-\frac{r^2}{r_7^2}\right)-...+\left(1-\frac{r_2}{r}-\frac{r^2}{r_1^2}\right)-\left(1+\frac{r_1}{r}+\frac{r^2}{r_{10}^2}\right), \end{aligned} \tag{2.6.7}$$

$$\begin{aligned} 1+\frac{r_d}{r}+\frac{\Lambda_d r^2}{3} &= 1+\frac{r_1+r_2+...+r_{10}}{r}+\left(\frac{1}{r_1^2}+\frac{1}{r_2^2}+...+\frac{1}{r_{10}^2}\right)r^2 = \\ &= \left(1+\frac{r_{10}}{r}+\frac{r^2}{r_9^2}\right)-\left(1-\frac{r_9}{r}-\frac{r^2}{r_8^2}\right)+\left(1+\frac{r_8}{r}+\frac{r^2}{r_7^2}\right)-...+\left(1+\frac{r_2}{r}+\frac{r^2}{r_1^2}\right)-\left(1-\frac{r_1}{r}-\frac{r^2}{r_{10}^2}\right). \end{aligned} \tag{2.6.8}$$

The result is a 5 metric with signature $(+ - - -)$:

$$
\begin{aligned}
ds_1^{(-)2} = &\left\{\begin{matrix} \left(1-\frac{r_{10}}{r}+\frac{r^2}{r_9^2}\right)-\left(1+\frac{r_9}{r}-\frac{r^2}{r_8^2}\right)+\left(1-\frac{r_8}{r}+\frac{r^2}{r_7^2}\right)-\left(1+\frac{r_7}{r}-\frac{r^2}{r_6^2}\right)+\left(1-\frac{r_6}{r}+\frac{r^2}{r_5^2}\right)- \\ -\left(1+\frac{r_5}{r}-\frac{r^2}{r_4^2}\right)+\left(1-\frac{r_4}{r}+\frac{r^2}{r_3^2}\right)-\left(1+\frac{r_3}{r}-\frac{r^2}{r_2^2}\right)+\left(1-\frac{r_2}{r}+\frac{r^2}{r_1^2}\right)-\left(1+\frac{r_1}{r}-\frac{r^2}{r_{10}^2}\right) \end{matrix}\right\} c^2dt^2 - \\
&-\left\{\begin{matrix} \left(1-\frac{r_{10}}{r}+\frac{r^2}{r_9^2}\right)-\left(1+\frac{r_9}{r}-\frac{r^2}{r_8^2}\right)+\left(1-\frac{r_8}{r}+\frac{r^2}{r_7^2}\right)-\left(1+\frac{r_7}{r}-\frac{r^2}{r_6^2}\right)+\left(1-\frac{r_6}{r}+\frac{r^2}{r_5^2}\right)- \\ -\left(1+\frac{r_5}{r}-\frac{r^2}{r_4^2}\right)+\left(1-\frac{r_4}{r}+\frac{r^2}{r_3^2}\right)-\left(1+\frac{r_3}{r}-\frac{r^2}{r_2^2}\right)+\left(1-\frac{r_2}{r}+\frac{r^2}{r_1^2}\right)-\left(1+\frac{r_1}{r}-\frac{r^2}{r_{10}^2}\right) \end{matrix}\right\}^{-1} dr^2 - \\
&-r^2\left(d\theta^2+\sin^2\theta\, d\varphi^2\right),
\end{aligned}
\tag{2.6.9}
$$

$$
\begin{aligned}
ds_2^{(-)2} = &\left\{\begin{matrix} \left(1+\frac{r_{10}}{r}-\frac{r^2}{r_9^2}\right)-\left(1-\frac{r_9}{r}+\frac{r^2}{r_8^2}\right)+\left(1+\frac{r_8}{r}-\frac{r^2}{r_7^2}\right)-\left(1-\frac{r_7}{r}+\frac{r^2}{r_6^2}\right)+\left(1+\frac{r_6}{r}-\frac{r^2}{r_5^2}\right)- \\ -\left(1-\frac{r_5}{r}+\frac{r^2}{r_4^2}\right)+\left(1+\frac{r_4}{r}-\frac{r^2}{r_3^2}\right)-\left(1-\frac{r_3}{r}+\frac{r^2}{r_2^2}\right)+\left(1+\frac{r_2}{r}-\frac{r^2}{r_1^2}\right)-\left(1-\frac{r_1}{r}+\frac{r^2}{r_{10}^2}\right) \end{matrix}\right\} c^2dt^2 - \\
&-\left\{\begin{matrix} \left(1+\frac{r_{10}}{r}-\frac{r^2}{r_9^2}\right)-\left(1-\frac{r_9}{r}+\frac{r^2}{r_8^2}\right)+\left(1+\frac{r_8}{r}-\frac{r^2}{r_7^2}\right)-\left(1-\frac{r_7}{r}+\frac{r^2}{r_6^2}\right)+\left(1+\frac{r_6}{r}-\frac{r^2}{r_5^2}\right)- \\ -\left(1-\frac{r_5}{r}+\frac{r^2}{r_4^2}\right)+\left(1+\frac{r_4}{r}-\frac{r^2}{r_3^2}\right)-\left(1-\frac{r_3}{r}+\frac{r^2}{r_2^2}\right)+\left(1+\frac{r_2}{r}-\frac{r^2}{r_1^2}\right)-\left(1-\frac{r_1}{r}+\frac{r^2}{r_{10}^2}\right) \end{matrix}\right\}^{-1} dr^2 - \\
&-r^2\left(d\theta^2+\sin^2\theta\, d\varphi^2\right),
\end{aligned}
\tag{2.6.10}
$$

$$
\begin{aligned}
ds_3^{(-)2} = &\left\{\begin{matrix} \left(1-\frac{r_{10}}{r}-\frac{r^2}{r_9^2}\right)-\left(1+\frac{r_9}{r}+\frac{r^2}{r_8^2}\right)+\left(1-\frac{r_8}{r}-\frac{r^2}{r_7^2}\right)-\left(1+\frac{r_7}{r}+\frac{r^2}{r_6^2}\right)+\left(1-\frac{r_6}{r}-\frac{r^2}{r_5^2}\right)- \\ -\left(1+\frac{r_5}{r}+\frac{r^2}{r_4^2}\right)+\left(1-\frac{r_4}{r}-\frac{r^2}{r_3^2}\right)-\left(1+\frac{r_3}{r}+\frac{r^2}{r_2^2}\right)+\left(1-\frac{r_2}{r}-\frac{r^2}{r_1^2}\right)-\left(1+\frac{r_1}{r}+\frac{r^2}{r_{10}^2}\right) \end{matrix}\right\} c^2dt^2 - \\
&-\left\{\begin{matrix} \left(1-\frac{r_{10}}{r}-\frac{r^2}{r_9^2}\right)-\left(1+\frac{r_9}{r}+\frac{r^2}{r_8^2}\right)+\left(1-\frac{r_8}{r}-\frac{r^2}{r_7^2}\right)-\left(1+\frac{r_7}{r}+\frac{r^2}{r_6^2}\right)+\left(1-\frac{r_6}{r}-\frac{r^2}{r_5^2}\right)- \\ -\left(1+\frac{r_5}{r}+\frac{r^2}{r_4^2}\right)+\left(1-\frac{r_4}{r}-\frac{r^2}{r_3^2}\right)-\left(1+\frac{r_3}{r}+\frac{r^2}{r_2^2}\right)+\left(1-\frac{r_2}{r}-\frac{r^2}{r_1^2}\right)-\left(1+\frac{r_1}{r}+\frac{r^2}{r_{10}^2}\right) \end{matrix}\right\}^{-1} dr^2 - \\
&-r^2\left(d\theta^2+\sin^2\theta\, d\varphi^2\right),
\end{aligned}
\tag{2.6.11}
$$

$$ds_4^{(-)2}=\left\{\begin{array}{l}\left(1+\frac{r_{10}}{r}+\frac{r^2}{r_9^2}\right)-\left(1-\frac{r_9}{r}-\frac{r^2}{r_8^2}\right)+\left(1+\frac{r_8}{r}+\frac{r^2}{r_7^2}\right)-\left(1-\frac{r_7}{r}-\frac{r^2}{r_6^2}\right)+\left(1+\frac{r_6}{r}+\frac{r^2}{r_5^2}\right)-\\-\left(1-\frac{r_5}{r}-\frac{r^2}{r_4^2}\right)+\left(1+\frac{r_4}{r}+\frac{r^2}{r_3^2}\right)-\left(1-\frac{r_3}{r}-\frac{r^2}{r_2^2}\right)+\left(1+\frac{r_2}{r}+\frac{r^2}{r_1^2}\right)-\left(1-\frac{r_1}{r}-\frac{r^2}{r_{10}^2}\right)\end{array}\right\}c^2dt^2-$$

$$-\left\{\begin{array}{l}\left(1+\frac{r_{10}}{r}+\frac{r^2}{r_9^2}\right)-\left(1-\frac{r_9}{r}-\frac{r^2}{r_8^2}\right)+\left(1+\frac{r_8}{r}+\frac{r^2}{r_7^2}\right)-\left(1-\frac{r_7}{r}-\frac{r^2}{r_6^2}\right)+\left(1+\frac{r_6}{r}+\frac{r^2}{r_5^2}\right)-\\-\left(1-\frac{r_5}{r}-\frac{r^2}{r_4^2}\right)+\left(1+\frac{r_4}{r}+\frac{r^2}{r_3^2}\right)-\left(1-\frac{r_3}{r}-\frac{r^2}{r_2^2}\right)+\left(1+\frac{r_2}{r}+\frac{r^2}{r_1^2}\right)-\left(1-\frac{r_1}{r}-\frac{r^2}{r_{10}^2}\right)\end{array}\right\}^{-1}dr^2-$$

$$-r^2\left(d\theta^2+\sin^2\theta d\varphi^2\right),$$

(2.6.12)

$$ds_5^{(-)2}=c^2dt^2-dr^2-r^2\left(d\theta^2+\sin^2\theta d\varphi^2\right). \tag{2.6.13}$$

Similarly, substitution of series (2.6.4) in the metric (2.5.10) through (2.5.13) affords metrics with the antipodal signature (– + + +):

$$ds_5^{(+)2}=-c^2dt^2+dr^2+r^2\left(d\theta^2+\sin^2\theta\,d\varphi^2\right). \tag{2.6.14}$$

$$ds_4^{(+)2}=-\left\{\begin{array}{l}\left(1+\frac{r_{10}}{r}+\frac{r^2}{r_9^2}\right)-\left(1-\frac{r_9}{r}-\frac{r^2}{r_8^2}\right)+\left(1+\frac{r_8}{r}+\frac{r^2}{r_7^2}\right)-\left(1-\frac{r_7}{r}-\frac{r^2}{r_6^2}\right)+\left(1+\frac{r_6}{r}+\frac{r^2}{r_5^2}\right)-\\-\left(1-\frac{r_5}{r}-\frac{r^2}{r_4^2}\right)+\left(1+\frac{r_4}{r}+\frac{r^2}{r_3^2}\right)-\left(1-\frac{r_3}{r}-\frac{r^2}{r_2^2}\right)+\left(1+\frac{r_2}{r}+\frac{r^2}{r_1^2}\right)-\left(1-\frac{r_1}{r}-\frac{r^2}{r_{10}^2}\right)\end{array}\right\}c^2dt^2+$$

$$+\left\{\begin{array}{l}\left(1+\frac{r_{10}}{r}+\frac{r^2}{r_9^2}\right)-\left(1-\frac{r_9}{r}-\frac{r^2}{r_8^2}\right)+\left(1+\frac{r_8}{r}+\frac{r^2}{r_7^2}\right)-\left(1-\frac{r_7}{r}-\frac{r^2}{r_6^2}\right)+\left(1+\frac{r_6}{r}+\frac{r^2}{r_5^2}\right)-\\-\left(1-\frac{r_5}{r}-\frac{r^2}{r_4^2}\right)+\left(1+\frac{r_4}{r}+\frac{r^2}{r_3^2}\right)-\left(1-\frac{r_3}{r}-\frac{r^2}{r_2^2}\right)+\left(1+\frac{r_2}{r}+\frac{r^2}{r_1^2}\right)-\left(1-\frac{r_1}{r}-\frac{r^2}{r_{10}^2}\right)\end{array}\right\}^{-1}dr^2+ \tag{2.6.15}$$

$$+r^2\left(d\theta^2+\sin^2\theta d\varphi^2\right),$$

$$ds_3^{(+)2}=-\left\{\begin{array}{l}\left(1-\frac{r_{10}}{r}-\frac{r^2}{r_9^2}\right)-\left(1+\frac{r_9}{r}+\frac{r^2}{r_8^2}\right)+\left(1-\frac{r_8}{r}-\frac{r^2}{r_7^2}\right)-\left(1+\frac{r_7}{r}+\frac{r^2}{r_6^2}\right)+\left(1-\frac{r_6}{r}-\frac{r^2}{r_5^2}\right)-\\-\left(1+\frac{r_5}{r}+\frac{r^2}{r_4^2}\right)+\left(1-\frac{r_4}{r}-\frac{r^2}{r_3^2}\right)-\left(1+\frac{r_3}{r}+\frac{r^2}{r_2^2}\right)+\left(1-\frac{r_2}{r}-\frac{r^2}{r_1^2}\right)-\left(1+\frac{r_1}{r}+\frac{r^2}{r_{10}^2}\right)\end{array}\right\}c^2dt^2+$$

$$+\left\{\begin{array}{l}\left(1-\frac{r_{10}}{r}-\frac{r^2}{r_9^2}\right)-\left(1+\frac{r_9}{r}+\frac{r^2}{r_8^2}\right)+\left(1-\frac{r_8}{r}-\frac{r^2}{r_7^2}\right)-\left(1+\frac{r_7}{r}+\frac{r^2}{r_6^2}\right)+\left(1-\frac{r_6}{r}-\frac{r^2}{r_5^2}\right)-\\-\left(1+\frac{r_5}{r}+\frac{r^2}{r_4^2}\right)+\left(1-\frac{r_4}{r}-\frac{r^2}{r_3^2}\right)-\left(1+\frac{r_3}{r}+\frac{r^2}{r_2^2}\right)+\left(1-\frac{r_2}{r}-\frac{r^2}{r_1^2}\right)-\left(1+\frac{r_1}{r}+\frac{r^2}{r_{10}^2}\right)\end{array}\right\}^{-1}dr^2+ \tag{2.6.16}$$

$$+r^2\left(d\theta^2+\sin^2\theta d\varphi^2\right),$$

$$ds_2^{(+)2} = \left\{ \begin{array}{l} \left(1+\frac{r_{10}}{r}-\frac{r^2}{r_9^2}\right)-\left(1-\frac{r_9}{r}+\frac{r^2}{r_8^2}\right)+\left(1+\frac{r_8}{r}-\frac{r^2}{r_7^2}\right)-\left(1-\frac{r_7}{r}+\frac{r^2}{r_6^2}\right)+\left(1+\frac{r_6}{r}-\frac{r^2}{r_5^2}\right)- \\ -\left(1-\frac{r_5}{r}+\frac{r^2}{r_4^2}\right)+\left(1+\frac{r_4}{r}-\frac{r^2}{r_3^2}\right)-\left(1-\frac{r_3}{r}+\frac{r^2}{r_2^2}\right)+\left(1+\frac{r_2}{r}-\frac{r^2}{r_1^2}\right)-\left(1-\frac{r_1}{r}+\frac{r^2}{r_{10}^2}\right) \end{array} \right\} c^2dt^2 +$$

$$+\left\{ \begin{array}{l} \left(1+\frac{r_{10}}{r}-\frac{r^2}{r_9^2}\right)-\left(1-\frac{r_9}{r}+\frac{r^2}{r_8^2}\right)+\left(1+\frac{r_8}{r}-\frac{r^2}{r_7^2}\right)-\left(1-\frac{r_7}{r}+\frac{r^2}{r_6^2}\right)+\left(1+\frac{r_6}{r}-\frac{r^2}{r_5^2}\right)- \\ -\left(1-\frac{r_5}{r}+\frac{r^2}{r_4^2}\right)+\left(1+\frac{r_4}{r}-\frac{r^2}{r_3^2}\right)-\left(1-\frac{r_3}{r}+\frac{r^2}{r_2^2}\right)+\left(1+\frac{r_2}{r}-\frac{r^2}{r_1^2}\right)-\left(1-\frac{r_1}{r}+\frac{r^2}{r_{10}^2}\right) \end{array} \right\}^{-1} dr^2 + \qquad (2.6.17)$$

$$+r^2\left(d\theta^2+\sin^2\theta d\varphi^2\right),$$

$$ds_1^{(+)2} = \left\{ \begin{array}{l} \left(1-\frac{r_{10}}{r}+\frac{r^2}{r_9^2}\right)-\left(1+\frac{r_9}{r}-\frac{r^2}{r_8^2}\right)+\left(1-\frac{r_8}{r}+\frac{r^2}{r_7^2}\right)-\left(1+\frac{r_7}{r}-\frac{r^2}{r_6^2}\right)+\left(1-\frac{r_6}{r}+\frac{r^2}{r_5^2}\right)- \\ -\left(1+\frac{r_5}{r}-\frac{r^2}{r_4^2}\right)+\left(1-\frac{r_4}{r}+\frac{r^2}{r_3^2}\right)-\left(1+\frac{r_3}{r}-\frac{r^2}{r_2^2}\right)+\left(1-\frac{r_2}{r}+\frac{r^2}{r_1^2}\right)-\left(1+\frac{r_1}{r}-\frac{r^2}{r_{10}^2}\right) \end{array} \right\} c^2dt^2 +$$

$$+\left\{ \begin{array}{l} \left(1-\frac{r_{10}}{r}+\frac{r^2}{r_9^2}\right)-\left(1+\frac{r_9}{r}-\frac{r^2}{r_8^2}\right)+\left(1-\frac{r_8}{r}+\frac{r^2}{r_7^2}\right)-\left(1+\frac{r_7}{r}-\frac{r^2}{r_6^2}\right)+\left(1-\frac{r_6}{r}+\frac{r^2}{r_5^2}\right)- \\ -\left(1+\frac{r_5}{r}-\frac{r^2}{r_4^2}\right)+\left(1-\frac{r_4}{r}+\frac{r^2}{r_3^2}\right)-\left(1+\frac{r_3}{r}-\frac{r^2}{r_2^2}\right)+\left(1-\frac{r_2}{r}+\frac{r^2}{r_1^2}\right)-\left(1+\frac{r_1}{r}-\frac{r^2}{r_{10}^2}\right) \end{array} \right\}^{-1} dr^2 + \qquad (2.6.18)$$

$$+r^2\left(d\theta^2+\sin^2\theta d\varphi^2\right).$$

We now consider what the radii r_k in the metric expressed by (2.6.5) through (2.6.18) may equal. It is natural to assume that in a fully geometrized physics only geometric constants must be present. These constants may include: R_v, the parametric radius of the Universe; and $l_c \approx c\,\Delta t \approx c \cdot 1$ sec $\approx 2{,}9\cdot 10^{10}$ cm that is, the distance traveled by the beam of light in a vacuum during a single time interval $\Delta t = 1$ second.

Assuming that the radii r_k in the metric (2.6.5) through (2.6.18) is estimated as the ratio

$$r_k \sim R_v^2/l_{ck}\ ,$$

where $l_{ck} = (2{,}9\cdot 10^{10})^{k}$ cm is the distance obtained by raising the number $2.9\cdot 10^{10}$ to the power of k. If we assume that $R_v \approx 10^{25}$ cm, we get the approximate relation

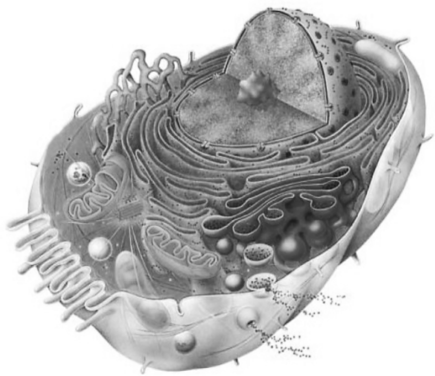
$$r_k \sim \frac{R_v^2}{l_{ck}} = \frac{10^{50}}{(2.9\cdot 10^{10})^{k}}\ \text{cm}, \qquad (2.6.19)$$

which implies a hierarchical sequence of the radii of the ten spheres.

Terminology: *Before proceeding, we must note that some of the entities here are analogues to, and sometimes overlapping with, measurements related to measurable subatomic particles, although the new entities are more general and not necessarily directly measurable, for the moment only appearing in calculations. (We leave aside the philosophical considerations as to whether all terms in a calculation must correspond to a real physical entity when the end result is just the same. The recurring debate on the "reality" of the wave function is an example that both sides have a basis to justify their positions. As well, many terms graduate from purely mathematical entities to representations of real entities, such as was the case with the positron, the neutrino, and countless other entities that we regard today as real).*

In this work, the names of the individual particles are put into guillemets, for exampel: «electron», «muon», etc. In this way metric-dynamic models of given local vacuum entities of Alsigna are clearly distinguished from the corresponding particles in the Standard Model and in String Theory.

The usual analysis breaks up an entity into sub-entities, which are then broken down in their turn, each layer using a different structure until one arrives at elementary particles. The structure proposed in this paper, on the other hand, is available at all levels, even for the elementary particles.



The terms for the constituents at one layer below the «particles», use one coined word: "particelle" (coined from "particle" and "organelle"), and three other terms: "scope", "outer shell ", and "abyss"; the usage of these latter three terms should be clearly distinguishable by the context from those of other contexts. In fact, it would be more useful to consider these as structures, applicable to a wide variety and scale of physical entities, than as particles. This difference is emphasized in the list below (6.20). The reader will immediately note in that list, whereas many of the numbers could correspond to directly measurable quantities, others clearly do not. For example, lengths are given that are beyond the range of measurement: bigger than the observable universe, and smaller than the Planck distance.

Of course, we could have left each r_k named simply "r_k" for respective values of k. However, we hope that the names assigned will serve as an aid to intuition, whereby one should not take the names any more literally Lengths r_2 through r_6 are within an order of magnitude of well-known physical measurements.

Furthermore, we do not use a zero length for any particle, because we do not really use particles in the classic sense. After all, particles are defined as stable local deformations of vacuum. We uses the word "particle" for convenience, although it is stable area of strong deformations and bound intra-vacuum currents.

With this preamble, we can now proceed to calculations using the approximate recurrence relation (6.19):

(6.20)

$r_1 \sim 3{,}4\cdot10^{39}$ cm: ~ «Universe» inner core ;

$r_2 \sim 1{,}2\cdot10^{29}$ cm: ~ «metagalaxy» inner core;

$r_3 \sim 4\cdot10^{18}$ cm: ~ «galaxy» inner core;

$r_4 \sim 1{,}4\cdot10^{8}$ cm: ~ «star» or «planet» inner core;

$r_5 \sim 4{,}9\cdot10^{-3}$ cm: ~ biological «cage» inner core;

$r_6 \sim 1{,}7\cdot10^{-13}$ cm: ~ core of an elementary «particle»;

$r_7 \sim 5{,}8\cdot10^{-24}$ cm: ~ core of an «protoquark»;

$r_8 \sim 2{,}1\cdot10^{-34}$ cm: ~ core of an «plankton»;

$r_9 \sim 7\cdot10^{-45}$ cm: ~ core of an «phytoplankton»;

$r_{10} \sim 2{,}4\cdot10^{-55}$ cm: ~ core of an «instanton».

The radii r_2, r_3, r_4 and r_5 are commensurate with the average radii of the nuclei of real spherical formations: metagalaxies, galaxies, stars (planets) and biological cells, and the radius r_6 practically coincided with the "classical radius" of an electron of $2{,}8\cdot10^{-13}$cm. Therefore, it is possible that the remaining radii r_1, r_7, r_8, r_9 and r_{10} of this sequence also correspond to the average radii of the spherical formations that inhabit the world.

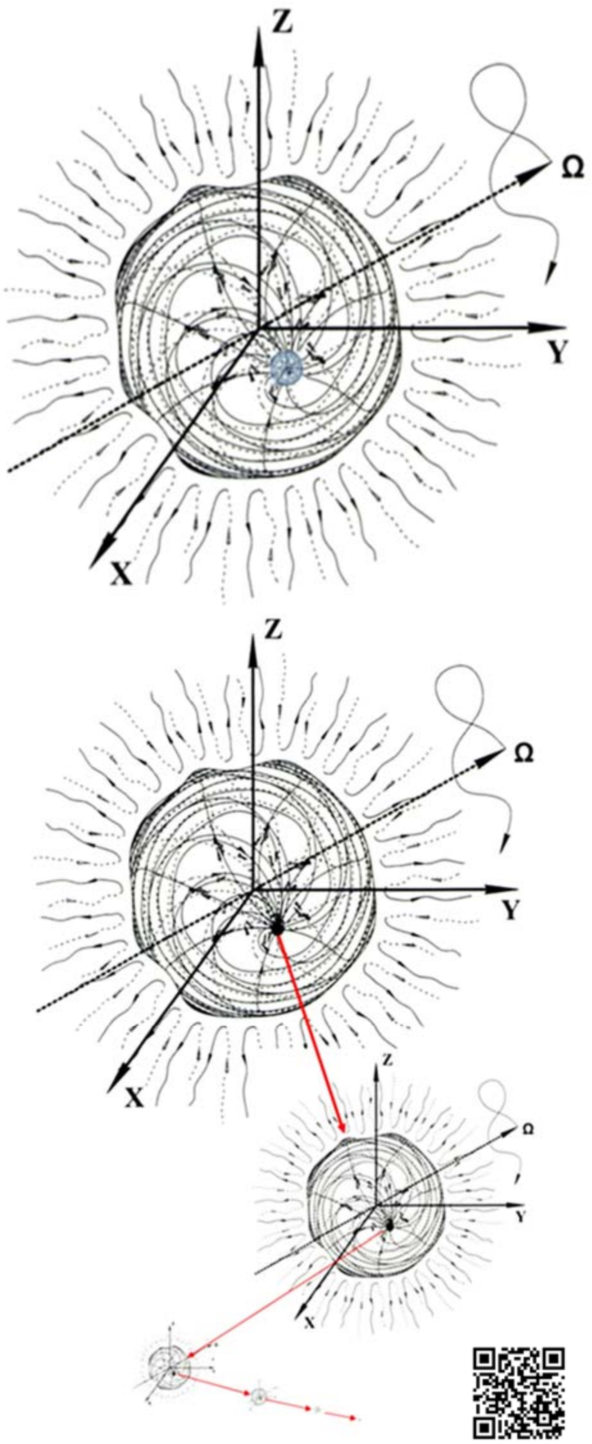


Fig. 6.2. Hierarchy of ten nested spherical vacuum formations

Fig. 2.6.2 a. Fractal illustration of the hierarchy of nested spherical vacuum formations

Metrics (2.6.9) through (2.6.18) are the solutions of the simplified to ten Λ_i-terms of Einstein's third vacuum equation (2.5.1):

$$R_{ik} - g_{ik}\sum_{k=1}^{10}\Lambda_k = 0\,, \qquad (2.6.21)$$

where
$$\sum_{k=1}^{10}\Lambda_k = \sum_{k=1}^{10}\frac{3}{r_k^2} + \sum_{k=1}^{10}\left(-\frac{3}{r_k^2}\right) = 0\,,$$

$$r_f = \sum_{k=1}^{10} r_k + \sum_{k=1}^{10}\left(-r_k\right) = 0.$$

In the hierarchy of the radii r_k (2.6.20), these solutions describe a sequence of nested spherical vacuum formations (Figures 2.6.1, 2.6.2).

For example, consider one of the vacuum degree of the hierarchy (2.6.20) with a radius r_6 ~$1{,}7\cdot10^{-13}$ cm corresponding to the characteristic size of the "core" of “elementary particles”. All other vacuum formations of the hierarchy considered here (2.6.20) are arranged similarly.

The radius of the core of such a formation is almost the same as the Thompson scattering length (aka the Lorenz radius). Despite the fact that the Thompson scattering length, 2,8·10^{-13}cm, is unrelated to the actual size of the electron, it is called the "classical radius of the electron". Since this length is the same order of magnitude of the value for the radius $r_6 \approx 1{,}7\cdot 10^{-13}$*cm of this formation, we find it fitting to dub the «particle» at this scale the «electron».*

In the metrics (2.6.9) through (2.6.12) will leave only those composed which contain radii $r_6 \sim 1{,}7\cdot 10^{-13}$cm. As a result, we obtain the following multilayer metric-dynamic model of «electron» (i.e. convex vacuum formation) with a core radius almost equal to "the classical radius of electron" $r_e \approx 2{,}8\cdot 10^{-13}$ cm:

«Electron» (2.6.22)

The «electron» is a convex multilayer vacuum formation
with signature $(+\,-\,-\,-)$
consisting of:

[a] The outer shell of the «electron»

in the interval [r_5, r_6] (Figure 2.6.3)

$$ds_1^{(-)2} = \left(1 - \frac{r_6}{r} + \frac{r^2}{r_5^2}\right)c^2dt^2 - \frac{dr^2}{\left(1 - \frac{r_6}{r} + \frac{r^2}{r_5^2}\right)} - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \qquad (2.6.23)$$

$$ds_2^{(-)2} = \left(1 + \frac{r_6}{r} - \frac{r^2}{r_5^2}\right)c^2dt^2 - \frac{dr^2}{\left(1 + \frac{r_6}{r} - \frac{r^2}{r_5^2}\right)} - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \qquad (2.6.24)$$

$$ds_3^{(-)2} = \left(1 - \frac{r_6}{r} - \frac{r^2}{r_5^2}\right)c^2dt^2 - \frac{dr^2}{\left(1 - \frac{r_6}{r} - \frac{r^2}{r_5^2}\right)} - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \qquad (2.6.25)$$

$$ds_4^{(-)2} = \left(1 + \frac{r_6}{r} + \frac{r^2}{r_5^2}\right)c^2dt^2 - \frac{dr^2}{\left(1 + \frac{r_6}{r} + \frac{r^2}{r_5^2}\right)} - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right); \qquad (2.6.26)$$

[b] The core of the «electron»

in the interval [r_6, r_7] (Figure 2.6.3)

$$ds_1^{(-)2} = \left(1 - \frac{r_7}{r} + \frac{r^2}{r_6^2}\right)c^2dt^2 - \frac{dr^2}{\left(1 - \frac{r_7}{r} + \frac{r^2}{r_6^2}\right)} - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \qquad (2.6.27)$$

$$ds_2^{(-)2} = \left(1 + \frac{r_7}{r} - \frac{r^2}{r_6^2}\right)c^2dt^2 - \frac{dr^2}{\left(1 + \frac{r_7}{r} - \frac{r^2}{r_6^2}\right)} - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \qquad (2.6.28)$$

$$ds_3^{(-)2} = \left(1 - \frac{r_7}{r} - \frac{r^2}{r_6^2}\right)c^2dt^2 - \frac{dr^2}{\left(1 - \frac{r_7}{r} - \frac{r^2}{r_6^2}\right)} - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \qquad (2.6.29)$$

$$ds_4^{(-)2} = \left(1 + \frac{r_7}{r} + \frac{r^2}{r_6^2}\right)c^2dt^2 - \frac{dr^2}{\left(1 + \frac{r_7}{r} + \frac{r^2}{r_6^2}\right)} - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right); \qquad (2.6.30)$$

[c] The scope of the «electron»

in the interval [0, ∞]

$$ds_5^{(-)2} = c^2dt^2 - dr^2 - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right). \qquad (2.6.31)$$

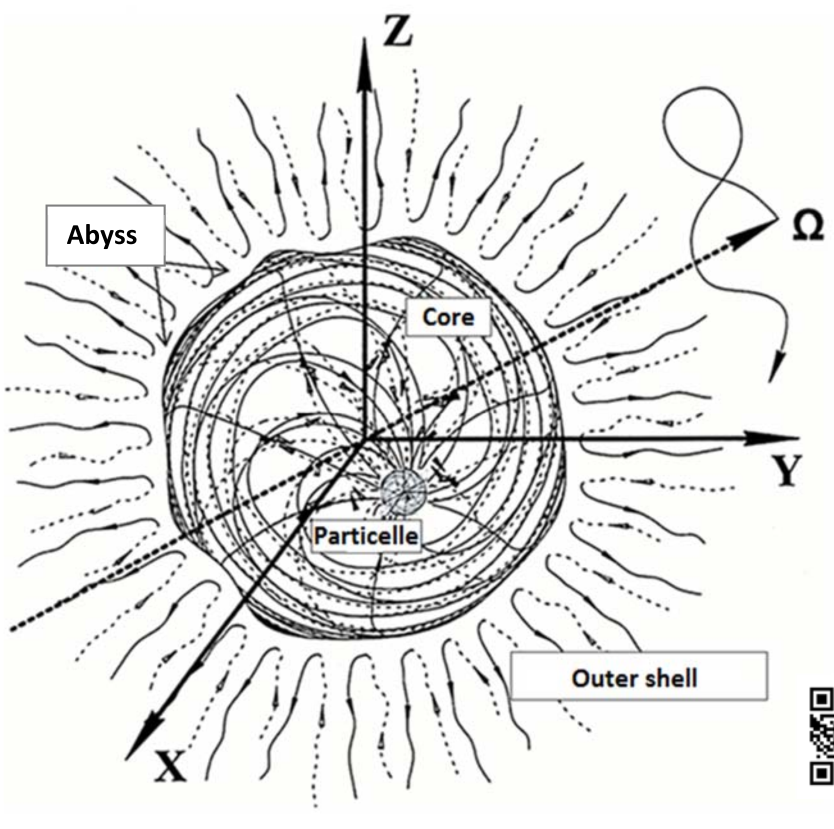


Fig. 2.6.3 Outer shell, abyss (*rakya*), core and the internal particelle of the elementary «particle»

Similarly, in the metrics (2.6.15) through (2.6.18) we leave only those terms that contain the radii r_6. As a result, we obtain the following metric-dynamic model of a conditionally concave vacuum formation, which we will call the «positron» (exact antipode to an «electron»):

«Positron» (2.6.32)

The «positron» is a concave vacuum formation
with the signature
$(-+++)$
consisting of:

[a] The outer shell of the «positron»

in the interval $[r_5, r_6]$
(Figure 2.6.3)

$$ds_1^{(+)2} = -\left(1-\frac{r_6}{r}+\frac{r^2}{r_5^2}\right)c^2dt^2 + \frac{dr^2}{\left(1-\frac{r_6}{r}+\frac{r^2}{r_5^2}\right)} + r^2\left(d\theta^2+\sin^2\theta\, d\varphi^2\right), \qquad (2.6.33)$$

$$ds_2^{(+)2} = -\left(1+\frac{r_6}{r}-\frac{r^2}{r_5^2}\right)c^2dt^2 + \frac{dr^2}{\left(1+\frac{r_6}{r}-\frac{r^2}{r_5^2}\right)} + r^2\left(d\theta^2+\sin^2\theta\, d\varphi^2\right), \qquad (2.6.34)$$

$$ds_3^{(+)2} = -\left(1-\frac{r_6}{r}-\frac{r^2}{r_5^2}\right)c^2dt^2 + \frac{dr^2}{\left(1-\frac{r_6}{r}-\frac{r^2}{r_5^2}\right)} + r^2\left(d\theta^2+\sin^2\theta\, d\varphi^2\right), \qquad (2.6.35)$$

$$ds_4^{(+)2} = -\left(1+\frac{r_6}{r}+\frac{r^2}{r_5^2}\right)c^2dt^2 + \frac{dr^2}{\left(1+\frac{r_6}{r}+\frac{r^2}{r_5^2}\right)} + r^2\left(d\theta^2+\sin^2\theta\, d\varphi^2\right), \qquad (2.6.36)$$

[b] The core of the «positron»

in the interval $[r_6, r_7]$ (Figure 2.6.3)

$$ds_1^{(+)2} = -\left(1-\frac{r_7}{r}+\frac{r^2}{r_6^2}\right)c^2dt^2 + \frac{dr^2}{\left(1-\frac{r_7}{r}+\frac{r^2}{r_6^2}\right)} + r^2\left(d\theta^2+\sin^2\theta\, d\varphi^2\right), \qquad (2.6.37)$$

$$ds_2^{(+)2} = -\left(1+\frac{r_7}{r}-\frac{r^2}{r_6^2}\right)c^2dt^2 + \frac{dr^2}{\left(1+\frac{r_7}{r}-\frac{r^2}{r_6^2}\right)} + r^2\left(d\theta^2+\sin^2\theta\, d\varphi^2\right), \qquad (2.6.38)$$

$$ds_3^{(+)2} = -\left(1-\frac{r_7}{r}-\frac{r^2}{r_6^2}\right)c^2dt^2 + \frac{dr^2}{\left(1-\frac{r_7}{r}-\frac{r^2}{r_6^2}\right)} + r^2\left(d\theta^2+\sin^2\theta\, d\varphi^2\right), \qquad (2.6.39)$$

$$ds_4^{(+)2} = -\left(1+\frac{r_7}{r}+\frac{r^2}{r_6^2}\right)c^2dt^2 + \frac{dr^2}{\left(1+\frac{r_7}{r}+\frac{r^2}{r_6^2}\right)} + r^2\left(d\theta^2+\sin^2\theta\, d\varphi^2\right), \qquad (2.6.40)$$

[c] The scope of the «positron»

in the interval [0, ∞]

$$ds_5^{(-)2} = c^2dt^2 - dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right); \qquad (2.6.41)$$

The sets of metrics (2.6.23) through (2.6.31) and (2.6.33) through (2.6.41) differ only in signature. That is, «electron» (2.6.22) and «positron» (2.6.32) are completely identical antipode copies of each other. If the «electron» is conventionally called the "convexity" of the vacuum extent, then the «positron» is exactly the same "concavity" of it.

Figure 2.6.3 shows a geometricized model of a spherical vacuum formation with subformations, using radii of the hierarchy (2.6.20).

Taking, for example, the «electron» (or its antipode, the «positron»), the formation represented in Figure 2.6.3 would have: a "core" with a radius $r_6 \sim 1{,}7\cdot10^{-13}$ cm; an inner particelle with a radius $r_7 \sim 5{,}8\cdot10^{-24}$ cm and an outer shell extending from $r_6 \sim 1{,}7\cdot10^{-13}$cm to $r_5 \sim 4{,}9\cdot10^{-3}$cm (or to $r_4 \sim 1{,}4\cdot10^{8}$ cm, or up to $r_3 \sim 4\cdot10^{18}$ cm, etc., depending on in which spherical formation there is an core of the «electron»).

In another case, for example, «planet» inner core has a radius $r_4 \sim 1{,}4\cdot10^{8}$ cm; its particelle has the radius $r_5 \sim 4{,}9\cdot10^{-3}$cm (or, $r_6 \sim 1{,}7\cdot10^{-13}$ cm, etc., depending on which spherical formation is found in the «planet» inner core) and the outer shell extends from $r_4 \sim 1{,}4\cdot10^{8}$ cm to $r_3 \sim 4\cdot10^{18}$ cm (or until $r_2 \sim 1{,}2\cdot10^{29}$ cm, or up to $r_1 \sim 3{,}4\cdot10^{39}$ cm).

The "scope" (2.6.31) or (2.6.41) of a spherical vacuum formation begins at its the center and ends at infinity. The scope represents a kind of memory of the undeformed portion of the considered vacuum area. It is almost as if it does not exist in the curved portion of the vacuum state, but according to equation (2.1.32), the relative elongation and deformation of the vacuum section cannot be determined without the $g_{ii}^{0(-)}$ of the scope.

The «abyss» (*rakya*) (Figure 2.6.3) is a spherical boundary between the core and the outer shell of any spherical vacuum formation.

2.7 Lucas-Fibonacci branches

We return to the series (2.5.2)

$$\Lambda_0 = \sum_{k=1}^{\infty} \Lambda_k = 3\sum_{k=1}^{\infty} \left(-1\right)^k \frac{N_k}{r_k^2} = 0\,. \qquad (2.7.1)$$

Among the many numerical sequences, the familiar Fibonacci sequence, 0,1,1,2,3,5,8,11... occupies a special place. It may be extended to the negative numbers, yielding the less familiar “negafibonacci numbers”: (2.7.2)

F–8	F–7	F–6	F–5	F–4	F–3	F–2	F–1	F0	F1	F2	F3	F4	F5	F6	F7	F8
–21	13	–8	5	–3	2	–1	1	0	1	1	2	3	5	8	13	21

We can also modify it to "seed" the beginning two numbers of the recursion, using 0 and –1 for the seeds, yielding

$$... 21,–13, 8, –5, 3,–2,1,–1, 0,–1,–1,–2,–3,–5,–8,–13,–21... \quad (2.7.3)$$

All of these follow the recursion relation

$$F_n = F_{n-1} + F_{n-2}.$$

We may now use the negafibonacci numbers for our sequence N_k in the series (2.7.1), labeling the n^{th} term in the sequence F_n for integer n, yielding

$$\Lambda_0 = \sum_{n=1}^{\infty} \Lambda_{nk} = 3\left(\sum_{n=-\infty}^{\infty} \frac{F_n}{r_k^2} + \sum_{k=-\infty}^{\infty} \frac{F_n^{'}}{r_k^2}\right) = 0 \,. \quad (2.7.4)$$

Also Lucas numbers can be used, which are defined by the recurrence formula

$$L_n=L_{n-1} +L_{n-2} \text{ for } L_0= 2 \text{ and } L_1= 1; \text{ or } L_n = \varphi^n + (1-\varphi)^n = \varphi^n + (-\varphi)^{-n}, \quad (2.7.5)$$

where the golden section Phi, $\varphi = \frac{1+\sqrt{5}}{2}$.

One example of a Lucas sequence occurs by using the values 2 and 1 for $n = 0, 1$:

$$L_n: \quad 2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, \ldots \quad (2.7.6)$$

In this case, (2.7.1) can take the form

$$\Lambda_0 = \sum_{n=1}^{\infty} \Lambda_{nk} = 3\left(\sum_{n=1}^{\infty} \frac{L_n}{r_k^2} + \sum_{n=1}^{\infty} \frac{-L_n^{'}}{r_k^2}\right) = 0 \,. \quad (2.7.7)$$

Taking into account the third Einstein field equations (2.7.4) and (2.7.7), the equation (2.5.1) can be written as

$$R_{ik} - g_{ik}\left(\sum_{n=1}^{\infty} \frac{3L_n}{r_k^2} + \sum_{n=1}^{\infty} \frac{-3L_n^{'}}{r_k^2} + \sum_{n=-\infty}^{\infty} \frac{3F_n}{r_k^2} + \sum_{n=-\infty}^{\infty} \frac{3F_n^{'}}{r_k^2}\right) = 0. \quad (2.7.8)$$

Since the conditions (2.7.4) and (2.7.7) are similar to (2.5.2), the solution of equation (2.7.8) will be similar to the solution of equation (2.5.1). The difference is that in the metrics (2.5.4) through (2.5.13) one should not substitute the series from (2.5.14), but rather, for the general case, the series

$$\Lambda_0 = \sum_{n=-\infty}^{\infty} \Lambda_n = \sum_{n=1}^{\infty} \frac{3L_n}{r_k^2} + \sum_{n=1}^{\infty} \frac{-3L_n^{'}}{r_k^2} + \sum_{n=-\infty}^{\infty} \frac{3F_n}{r_k^2} + \sum_{n=-\infty}^{\infty} \frac{3F_n^{'}}{r_k^2} = 0. \quad (2.7.9)$$

It is necessary to expect that the vacuum equations may include the Fibonacci numbers F_n, the Lucas numbers L_n and φ (the golden section), as they contribute to the harmony of so many other phenomena in nature. We follow up on this expectation.

Combining the results of this and previous points, we arrive at the following model of the physical universe: the hierarchical sequence of ten spheres with radii r_k (2.6.20) acts as a "trunk" and the solutions of equation (2.7.8) look like Lucas-Fibonacci branches radiating in all directions from this grand trunk.

Now we may ponder the following question. If the right sides of the Einstein field equations (2.1.6), (2.2.7) and (2.4.8) are equal to zero, leading to a state with no mass, what, then, fills the void?

In the framework developed here, this void is filled with a variety of spherical convex and concave vacuum formations with different radii (see Figures 2.6.1, 2.6.2 and 2.6.2 *a*), which interact with each other by means of vacuum currents. This is, however, not ether or Descartes' vortices, as we shall outline in the following outline.

Current interactions (electromagnetic, nuclear, and gravitational) between sphere vacuum formations of different scales are described in Chapters 5, 8, and 9.

2.8 The elements of the Algebra of signatures

We return to the metrics (2.1.16) and (2.1.19), which for brevity can be represented in a Cartesian coordinate system:

$$ds^{(+---)2} = c^2dt^2 - dx^2 - dy^2 - dz^2 = x_0^2 - x_1^2 - x_2^2 - x_3^2 = 0 \text{ with signature } (+---), \quad (2.8.1)$$

$$ds^{(-+++)2} = -c^2dt^2 + dx^2 + dy^2 + dz^2 = -x_0^2 + x_1^2 + x_2^2 + x_3^2 = 0 \text{ with signature } (-+++). \quad (2.8.2)$$

Here we use the following conventions:

$$s^{(+---)2} = ds^{(-)2},\quad s^{(-+++)2} = ds^{(+)2},\quad x_0^2 = c^2dt^2,\quad x_1^2 = dx^2,\quad x_2^2 = dy^2,\quad x_3^2 = dz^2. \quad (2.8.3)$$

These metrics are solutions at the same time all three vacuum equations (2.1.6), (2.2.7) and (2.4.8).

In addition to the metrics (2.8.1) and (2.8.2) with signatures $(+---)$ and $(-+++)$, 14 other possible metrics can be written with the corresponding signatures {see (1.11.1) and (1.13.7)}

$$\begin{aligned}
s^{(++++)2} &= x_0^2 + x_1^2 + x_2^2 + x_3^2 = 0\\
s^{(---+)2} &= -x_0^2 - x_1^2 - x_2^2 + x_3^2 = 0\\
s^{(+--+)2} &= x_0^2 - x_1^2 - x_2^2 + x_3^2 = 0\\
s^{(--+-)2} &= -x_0^2 - x_1^2 + x_2^2 - x_3^2 = 0\\
s^{(-+--)2} &= -x_0^2 + x_1^2 - x_2^2 - x_3^2 = 0\\
s^{(+-+-)2} &= x_0^2 - x_1^2 + x_2^2 - x_3^2 = 0\\
\underline{s^{(++--)2}} &\underline{= x_0^2 + x_1^2 - x_2^2 - x_3^2 = 0}\\
s^{(+---)2} &= x_0^2 - x_1^2 - x_2^2 - x_3^2 = 0
\end{aligned} \quad (2.8.4)
\qquad
\begin{aligned}
s^{(----)2} &= -x_0^2 - x_1^2 - x_2^2 - x_3^2 = 0\\
s^{(+++-)2} &= x_0^2 + x_1^2 + x_2^2 - x_3^2 = 0\\
s^{(-++-)2} &= -x_0^2 + x_1^2 + x_2^2 - x_3^2 = 0\\
s^{(++-+)2} &= x_0^2 + x_1^2 - x_2^2 + x_3^2 = 0\\
s^{(+-++)2} &= x_0^2 - x_1^2 + x_2^2 + x_3^2 = 0\\
s^{(-+-+)2} &= -x_0^2 + x_1^2 - x_2^2 + x_3^2 = 0\\
\underline{s^{(--++)2}} &\underline{= -x_0^2 - x_1^2 + x_2^2 + x_3^2 = 0}\\
s^{(-+++)2} &= -x_0^2 + x_1^2 + x_2^2 + x_3^2 = 0
\end{aligned} \quad (2.8.5)$$

Operations on the metrics (2.8.4) and (2.8.5) will be carried out componentwise, so we will call such aggregate metrics "ranks" (*see Chapter 1*).

Instead of the uniform terms in the ranks (2.8.4) and (2.8.5) being summed up directly, they can be summed up using only signs preceding these terms. So for brevity, instead of ranks (2.8.4) and (2.8.5), we can use the following equivalent ranks:

$$
\begin{array}{lcll}
(+\ +\ +\ +) & + & (-\ -\ -\ -) & = 0 \\
(-\ -\ -\ +) & + & (+\ +\ +\ -) & = 0 \\
(+\ -\ -\ +) & + & (-\ +\ +\ -) & = 0 \\
(-\ -\ +\ -) & + & (+\ +\ -\ +) & = 0 \\
(+\ +\ -\ -) & + & (-\ -\ +\ +) & = 0 \\
(-\ +\ -\ -) & + & (+\ -\ +\ +) & = 0 \\
\underline{(+\ -\ +\ -)} & + & \underline{(-\ +\ -\ +)} & = 0 \\
(+\ -\ -\ -)_{+} & + & (-\ +\ +\ +)_{+} & = 0\ ,
\end{array}
\qquad (2.8.6)
$$

The subscripted sign after the brackets $(...)_{+}$ indicates what operation is done with the numbers corresponding to the characters in the columns and/or rows; that is, $(...)_{+}$ for addition, $(...)_{-}$ for subtraction, $(...)_{/}$ for division and $(...)_{\times}$ for multiplication (see Definition 1.10.2). Although the other operations could be also defined componentwise, excluding division by zero, we shall not do so here, as presently we are only concerned with addition.

The metrics with the above features, as ranked in (2.8.4) and (2.8.5), are not solutions of the Einstein field equations (2.1.6), (2.2.7) and (2.4.8). This can be verified by direct substitution of the metric tensor components of these metrics in the corresponding equations.

None of the metrics above the line, i.e. in the numerators of the ranks (2.8.4) and (2.8.5), are solutions of the Einstein field equations (2.1.6), (2.2.7) and (2.4.8). This can be verified by direct substitution of the metric tensor components of these metrics in these equations.

However, regard the result from, for example, summing (as earlier explained) the first seven metrics of the ranking (2.8.4); it is the metric with signature $(+\,-\,-\,-)$: $s^{(+\,-\,-\,-)2} = x_0^2 - x_1^2 - x_2^2 - x_3^2 = 0$. (In order to make this calculation, one can simply add up the respective columns.)

Similarly, the sum of the first seven metrics ranked by (2.8.5) is wound with the opposite metric signature $(-\,+\,+\,+)$: $s^{(-\,+\,+\,+)2} = -x_0^2 + x_1^2 + x_2^2 + x_3^2 = 0$.

Therefore, vertically summing up the seven metrics of (2.8.4) and/or (2.8.5), leads to solutions of the Einstein field equations (2.1.6), (2.2.7) and (2.4.8):

(2.8.7)

$$ds^{(+---)2} = ds^{(++++)2} + ds^{(---+)2} + ds^{(+--+)2} + ds^{(--+-)2} + ds^{(++--)2} + ds^{(-+--)2} + ds^{(+-+-)2},$$
$$ds^{(-+++)2} = ds^{(----)2} + ds^{(+++-)2} + ds^{(-++-)2} + ds^{(++-+)2} + ds^{(--++)2} + ds^{(+-++)2} + ds^{(-+-+)2}.$$

The same is true of horizontal sums of the above. For example,

$$ds^{(+--+)2} + ds^{(-++-)2} = 0\cdot c^2dt^2 + 0\cdot dr^2 + 0\cdot d\theta^2 + 0\cdot \sin^2\theta\, d\varphi^2 = ds^{(0\,0\,0\,0)2}. \qquad (2.8.8)$$

In addition, the sum of all 16 metrics of (2.8.4) and (2.8.5) is a solution of the given vacuum equations

$$\begin{aligned} ds_\Sigma{}^2 = \; & ds^{(+---)2} + ds^{(++++)2} + ds^{(---+)2} + ds^{(+--+)2} + \\ + \; & ds^{(--+-)2} + ds^{(++--)2} + ds^{(-+--)2} + ds^{(+-+-)2} + \\ + \; & ds^{(-+++)2} + ds^{(----)2} + ds^{(+++-)2} + ds^{(-++-)2} + \\ + \; & ds^{(++-+)2} + ds^{(--++)2} + ds^{(+-++)2} + ds^{(-+-+)2} = ds^{(0\,0\,0\,0)2} = 0. \end{aligned} \qquad (2.8.9)$$

An equivalent representation of a signature of expression (2.8.9) has the form

$$\begin{aligned} & (+---) + (++++) + (---+) + (+--+) + \\ + \; & (--+-) + (++--) + (-+--) + (+-+-) + \\ + \; & (-+++) + (----) + (+++-) + (-++-) + \\ + \; & (++-+) + (--++) + (+-++) + (-+-+) = \{0\,0\,0\,0\}. \end{aligned} \qquad (2.8.10)$$

A structure based on these ranks takes the form of “vacuum conditions":

$$\begin{array}{lcccl} 0 = & \underline{(0\;\;0\;\;0\;\;0)} & + & \underline{(0\;\;0\;\;0\;\;0)} & = 0 \\ 0 = & (+\;\;+\;\;+\;\;+) & + & (-\;\;-\;\;-\;\;-) & = 0 \\ 0 = & (-\;\;-\;\;-\;\;+) & + & (+\;\;+\;\;+\;\;-) & = 0 \\ 0 = & (+\;\;-\;\;-\;\;+) & + & (-\;\;+\;\;+\;\;-) & = 0 \\ 0 = & (-\;\;-\;\;+\;\;-) & + & (+\;\;+\;\;-\;\;+) & = 0 \\ 0 = & (+\;\;+\;\;-\;\;-) & + & (-\;\;-\;\;+\;\;+) & = 0 \\ 0 = & (-\;\;+\;\;-\;\;-) & + & (+\;\;-\;\;+\;\;+) & = 0 \\ 0 = & (+\;\;-\;\;+\;\;-) & + & (-\;\;+\;\;-\;\;+) & = 0 \\ 0 = & \underline{(-\;\;+\;\;+\;\;+)} & + & \underline{(+\;\;-\;\;-\;\;-)} & = 0 \\ 0 = & (0\;\;0\;\;0\;\;0)_+ & + & (0\;\;0\;\;0\;\;0)_+ & = 0 \end{array}$$

(2.8.11)

This process could be called "splitting of zeros" (*see Chapter 1, Definition 1.12.1*).

The seventeen signatures (2.8.10) form a structure as indicated in the above introduction to the Algebra of signatures. A further structure which is developed in the cited references can be created by adding the Kronecker product and using the formation of the sixteen non-zero signatures of the ranked (2.8.11) in the anti-symmetric matrix resulting from the square using the Kronecker product of a 2 × 2 matrix of binary signatures [22]:

Fig. 2.8.1. Fractal illustration of the superposition and interlacing of sixteen 4-dimensional spaces with different signatures (topologies)

$$\begin{pmatrix} (++) & (+-) \\ (-+) & (--) \end{pmatrix}^{\otimes 2} = \begin{pmatrix} (++++) & (+++-) & (+-++) & (+-+-) \\ (++-+) & (-+++) & (+--+) & (+---) \\ (-+++) & (-++-) & (--++) & (--+-) \\ (-+-+) & (-+--) & (---+) & (----) \end{pmatrix} \qquad (2.8.12)$$

We shall not follow up on this possibility in this paper; the reader is referred to the papers alluded to above.

According to the classification of Felix Klein, quadratic forms (2.8.4) and (2.8.5) are divided into three topological classes [29] (*see Chapter 1*):

1st class: quadratic forms (metrics), the signatures of which are composed of four identical characters:

$$x_0^2 + x_1^2 + x_2^2 + x_3^2 = 0 \quad (+\,+\,+\,+) \qquad (2.8.13)$$

$$-x_0^2 - x_1^2 - x_2^2 - x_3^2 = 0 \quad (-\,-\,-\,-) \qquad (2.8.14)$$

represent a "null" 4-metric space. In these spaces, there is only one actual point that is at the beginning of the light cone. All other terms of these extents are imaginary. In fact, in this case the metric (2.8.13) does not describe a positive length but rather a single point (which we will term a "white" point); and the metric (2.8.14) describes a single anti-point (which we shall term a "black" point).

2nd class: metrics, whose signatures are composed of three identical symbols and one of the opposite:

$$\left.\begin{array}{ll}
-x_0^2 - x_1^2 - x_2^2 + x_3^2 = 0 & (- - - +) \\
-x_0^2 - x_1^2 + x_2^2 - x_3^2 = 0 & (- - + -) \\
-x_0^2 + x_1^2 - x_2^2 - x_3^2 = 0 & (- + - -) \\
x_0^2 - x_1^2 - x_2^2 - x_3^2 = 0 & (+ - - -) \\
x_0^2 + x_1^2 + x_2^2 - x_3^2 = 0 & (+ + + -) \\
x_0^2 + x_1^2 - x_2^2 + x_3^2 = 0 & (+ + - +) \\
x_0^2 - x_1^2 + x_2^2 + x_3^2 = 0 & (+ - + +) \\
-x_0^2 + x_1^2 + x_2^2 + x_3^2 = 0 & (- + + +)
\end{array}\right\} \quad (2.8.15)$$

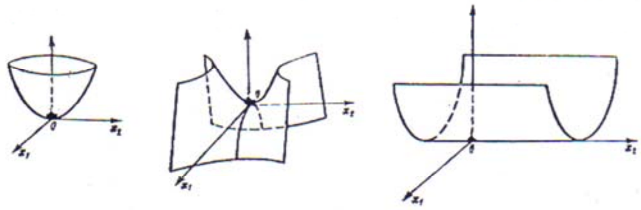
is an oval surface [29]: a) ellipsoid; b) elliptic paraboloid; c) two-sheeted hyperboloid (elliptic hyperboloid).

<u>3rd class</u>: metrics, the signatures of which are composed of two positive and two negative signs:

$$\left.\begin{array}{ll}
x_0^2 - x_1^2 - x_2^2 + x_3^2 = 0 & (+ - - +) \\
x_0^2 + x_1^2 - x_2^2 - x_3^2 = 0 & (+ + - -) \\
x_0^2 - x_1^2 + x_2^2 - x_3^2 = 0 & (+ - + -) \\
-x_0^2 + x_1^2 + x_2^2 - x_3^2 = 0 & (- + + -) \\
-x_0^2 - x_1^2 + x_2^2 + x_3^2 = 0 & (- - + +) \\
-x_0^2 + x_1^2 - x_2^2 + x_3^2 = 0 & (- + - +)
\end{array}\right\} \quad (2.8.16)$$

These represent a variety of options for annular surfaces (Klein 2004): (a) single-band hyperboloids; (b) hyperbolic paraboloids.

A simplified illustration of the signature due to its topology of the 2-dimensional region is shown in Figure 2.8.1.



a) *sign* (+ +); b) *sign* (– +); c) *sign* (+ 0)

$z = x_1^2 + x_2^2$ $z = x_2^2 - x_1^2$ $z = x_1^2$

Fig. 2.8.1 Signature of the metric connection with the topology of 2-dimensional length [29]

Such an additive overlay (or "atlas") of a 7-metric space with metrics (2.8.4) and (2.8.5) leads to the Ricci-flat spaces with the total metrics (2.8.1) and (2.8.2). Such a 7-sheeted atlas is very similar to the Ricci-flat 10-dimensional Calabi-Yau space.

Stability can only be:

- a convex vacuum formation, described by a metric with signature (+ – – –),
- a concave vacuum formation described by a metric with signature (– + + +),
- a “flat" vacuum formation, described by a metric with signature (0 0 0 0).

All the other 14 metrics (2.8.4) and (2.8.5) with the signatures of the numerators are ranked by (2.8.6)

$$\begin{array}{ll} (+\ +\ +\ +) & (-\ -\ -\ -) \\ (-\ -\ -\ +) & (+\ +\ +\ -) \\ (+\ -\ -\ +) & (-\ +\ +\ -) \\ (-\ -\ +\ -) & (+\ +\ -\ +) \\ (+\ +\ -\ -) & (-\ -\ +\ +) \\ (-\ +\ -\ -) & (+\ -\ +\ +) \\ (+\ -\ +\ -) & (-\ +\ -\ +) \end{array} \qquad (2.8.17)$$

describe various types of "convex-concave" states. The corresponding regions of the vacuum may not be stable, since metric data cannot be solutions of vacuum equations. They can occur as temporary complex distortions of a local vacuum area, but after some time they disappear or turn into other types of fluctuations with other signatures (or topologies).

However, if the additive superposition (combination) of several metric spaces with signatures (topologies) (2.8.17) in the amount results in an average "convex" vacuum formations with the signature (+ – – –), or on average "concave" formation of the vacuum with the signature (– + + +), or to the average "flat" vacuum education with signature (0 0 0 0), such a vacuum formation may be stable.

2.9 The «proton» & «antiproton»

Solutions of Einstein field equations (2.1.6), (2.2.7) and (2.4.8) lead not only to aggregate metrics (2.8.4) and (2.8.5), but, for example, also to additive combinations of metrics:

$$\begin{array}{ll} s^{(---+)2} = -x_0^2 - x_1^2 - x_2^2 + x_3^2 = 0 \\ s^{(+-+-)2} = x_0^2 - x_1^2 + x_2^2 - x_3^2 = 0 & (2.9.1) \\ \underline{s^{(++--)2} = x_0^2 + x_1^2 - x_2^2 - x_3^2 = 0} \\ s^{(+---)2} = x_0^2 - x_1^2 - x_2^2 - x_3^2 = 0 \end{array} \qquad \begin{array}{ll} s^{(+++-)2} = x_0^2 + x_1^2 + x_2^2 - x_3^2 = 0 \\ s^{(-+-+)2} = -x_0^2 + x_1^2 - x_2^2 + x_3^2 = 0 & (2.9.2) \\ \underline{s^{(--++)2} = -x_0^2 - x_1^2 + x_2^2 + x_3^2 = 0} \\ s^{(-+++)2} = -x_0^2 + x_1^2 + x_2^2 + x_3^2 = 0 \end{array}$$

There are three possibilities for the average convex of vacuum formation, which can be represented in an equivalent form:

$$\begin{array}{l} (- - - +) \\ (+ - + -) \\ \underline{(+ + - -)} \\ (+ - - -)_+ \end{array} \ (2.9.3) \qquad \begin{array}{l} (- - + -) \\ (+ + - -) \\ \underline{(+ - - +)} \\ (+ - - -)_+ \end{array} \ (2.9.4) \qquad \begin{array}{l} (- + - -) \\ (+ - - +) \\ \underline{(+ - + -)} \\ (+ - - -)_+ \end{array} \ (2.9.5)$$

and three possibilities for the average concave of vacuum formation:

$$\begin{array}{l} (+ + + -) \\ (- + - +) \\ \underline{(- - + +)} \\ (- + + +)_+ \end{array} \ (2.9.6) \qquad \begin{array}{l} (+ + - +) \\ (- - + +) \\ \underline{(- + + -)} \\ (- + + +)_+ \end{array} \ (2.9.7) \qquad \begin{array}{l} (+ - + +) \\ (- + + -) \\ \underline{(- + - +)} \\ (- + + +)_+ \end{array} \ (2.9.8)$$

Recall that the metrics (2.8.1) and (2.8.2) are special (limiting) cases of all other metrics (2.2.8) through (2.2.11) and (2.2.13) through (2.2.16) are solutions of the second vacuum equations (2.2.7). Therefore, the mathematical techniques, outlined by the author of the Algebra of Signatures as explained above, apply to all these solutions.

We will enter ideas of «quarks». To do this, we write the ranks (2.9.3) through (2.9.8) as follows:

$$\begin{array}{l} d_r^+(+ + + -) \\ u_g^-(- + - +) \\ u_b^-\underline{(- - + +)} \\ p_1^+(- + + +)_+ \end{array} \ (2.9.9) \qquad \begin{array}{l} d_g^+(+ + - +) \\ u_b^-(- - + +) \\ u_r^-\underline{(- + + -)} \\ p_2^+(- + + +)_+ \end{array} \ (2.9.10) \qquad \begin{array}{l} d_b^+(+ - + +) \\ u_r^-(- + + -) \\ u_g^-\underline{(- + - +)} \\ p_3^+(- + + +)_+ \end{array} \ (2.9.11)$$

where p_i^+ are three different states of an «proton» (i = 1, 2, 3).

$$\begin{array}{l} d_r^-(- - - +) \\ u_g^+(+ - + -) \\ u_b^+\underline{(+ + - -)} \\ p_1^-(+ - - -)_+ \end{array} \ (2.9.12) \qquad \begin{array}{l} d_g^-(- - + -) \\ u_b^+(+ + - -) \\ u_r^+\underline{(+ - - +)} \\ p_2^-(+ - - -)_+ \end{array} \ (2.9.13) \qquad \begin{array}{l} d_b^-(- + - -) \\ u_r^+(+ - - +) \\ u_g^+\underline{(+ - + -)} \\ p_3^-(+ - - -)_+ \end{array} \ (2.9.14)$$

where p_i^- are three different states of an «antiproton».

The sets of ten kinds of metrics (2.6.22) with the appropriate signatures from the matrix (2.8.12) will be termed as follows:

$$\begin{array}{llll}
\text{10 metrics}^{1} \text{ of the form (2.6.22) with signature } (+\,+\,+\,-) & : \text{ red} & d_r^{+}\text{-«quark»;} & \\
\text{10 metrics of the form (2.6.22) with signature } (+\,+\,-\,+) & : \text{ green} & d_g^{+}\text{-«quark»;} & (2.9.15) \\
\text{10 metrics of the form (2.6.22) with signature } (+\,-\,+\,+) & : \text{ blue} & d_b^{+}\text{-«quark»,} &
\end{array}$$

$$\begin{array}{llll}
\text{10 metrics of the form (2.6.22) with signature } (-\,-\,-\,+) & : \text{ red} & d_r^{-}\text{-«antiquark»;} & \\
\text{10 metrics of the form (2.6.22) with signature } (-\,-\,+\,-) & : \text{ green} & d_g^{-}\text{-«antiquark»;} & (2.9.16) \\
\text{10 metrics of the form (2.6.22) with signature } (-\,+\,-\,-) & : \text{ blue} & d_b^{-}\text{-«antiquark»,} &
\end{array}$$

$$\begin{array}{llll}
\text{10 metrics of the form (2.6.22) with signature } (+\,-\,-\,+) & : \text{ red} & u_r^{+}\text{-«quark»;} & \\
\text{10 metrics of the form (2.6.22) with signature } (+\,-\,+\,-) & : \text{ green} & u_g^{+}\text{-«quark»;} & (2.9.17) \\
\text{10 metrics of the form (2.6.22) with signature } (+\,+\,-\,-) & : \text{ blue} & u_b^{+}\text{-«quark».} &
\end{array}$$

$$\begin{array}{llll}
\text{10 metrics of the form (2.6.22) with signature } (-\,+\,+\,-) & : \text{ red} & u_r^{-}\text{-«antiquark»;} & \\
\text{10 metrics of the form (2.6.22) with signature } (-\,+\,-\,+) & : \text{ green} & u_g^{-}\text{-«ntiquark»;} & (2.9.18) \\
\text{10 metrics of the form (2.6.22) with signature } (-\,-\,+\,+) & : \text{ blue} & u_b^{-}\text{-«antiquark».} &
\end{array}$$

In this case, the three «proton» states of and three «antiproton» states may be represented as

$$p_1^{+} = u_g^{-}u_b^{-}d_r^{+}, \quad p_2^{+} = u_r^{-}u_b^{-}d_g^{+}, \quad p_3^{+} = u_g^{-}u_r^{-}d_b^{+}, \tag{2.9.19}$$

$$p_1^{-} = u_g^{-}u_b^{-}d_r^{+}, \quad p_2^{-} = u_r^{-}u_b^{-}d_g^{+}, \quad p_3^{-} = u_g^{-}u_r^{-}d_b^{+}, \tag{2.9.20}$$

similar to the notation and composition of the proton and antiproton in the Standard Model and in quantum chromodynamics. However, within the framework of the Algebra of Signatures, the «proton» and «antiproton» consist of «quarks» and «antiquarks», which allows us to outline ways to solve the problem of the coexistence of matter and antimatter. In addition, metric-dynamic models given by the Algebra of Signatures are obtained in a more straightforward and informative way. In addition, the metric-dynamic models of the Algebra of Signatures are much more visual and informative than the models of quantum chromodynamics. For example, consider a multilayered metric-dynamic model of the «proton» in the state (2.9.9):

---

[1] 10 metrics are of the form (2.6.22), because the scope (2.6.31), as well as the core, are related to the outer shell. In this way, 5 metrics describe the core, and 5 metrics describe the outer shell, to make up the total of 10 metrics.

«Proton» (2.9.21)

On the average, this is a concave multilayer vacuum formation
with a total (average) signature (2.9.9)

(– + + +),

consisting of:

[a] d_r^+-«quark» (2.9.22)

with signature

(+ + + –)

[a][i] The outer shell of the d_r^+-«quark»

in the interval [r_5, r_6]

(Figure 2.9.1):

$$ds_1^{(+++-)2} = \left(1 - \frac{r_6}{r} + \frac{r^2}{r_5^2}\right)c^2dt^2 + \frac{dr^2}{\left(1 - \frac{r_6}{r} + \frac{r^2}{r_5^2}\right)} + r^2d\theta^2 - r^2\sin^2\theta\, d\varphi^2,$$

$$ds_2^{(+++-)2} = \left(1 + \frac{r_6}{r} - \frac{r^2}{r_5^2}\right)c^2dt^2 + \frac{dr^2}{\left(1 + \frac{r_6}{r} - \frac{r^2}{r_5^2}\right)} + r^2d\theta^2 - r^2\sin^2\theta\, d\varphi^2,$$

$$ds_3^{(+++-)2} = \left(1 - \frac{r_6}{r} - \frac{r^2}{r_5^2}\right)c^2dt^2 + \frac{dr^2}{\left(1 - \frac{r_6}{r} - \frac{r^2}{r_5^2}\right)} + r^2d\theta^2 - r^2\sin^2\theta\, d\varphi^2,$$

$$ds_4^{(+++-)2} = \left(1 + \frac{r_6}{r} + \frac{r^2}{r_5^2}\right)c^2dt^2 + \frac{dr^2}{\left(1 + \frac{r_6}{r} + \frac{r^2}{r_5^2}\right)} + r^2d\theta^2 - r^2\sin^2\theta\, d\varphi^2;$$

[a][ii] The core of the d_r^+-«quark» (2.9.23)

in the interval [r_6, r_7]

(Figure 2.9.1)

$$ds_1^{(+++-)2} = \left(1 - \frac{r_7}{r} + \frac{r^2}{r_6^2}\right)c^2dt^2 + \frac{dr^2}{\left(1 - \frac{r_7}{r} + \frac{r^2}{r_6^2}\right)} + r^2d\theta^2 - r^2\sin^2\theta\, d\varphi^2,$$

$$ds_2^{(+++-)2} = \left(1 + \frac{r_7}{r} - \frac{r^2}{r_6^2}\right)c^2dt^2 + \frac{dr^2}{\left(1 + \frac{r_7}{r} - \frac{r^2}{r_6^2}\right)} + r^2d\theta^2 - r^2\sin^2\theta\, d\varphi^2,$$

$$ds_3^{(+++-)2} = \left(1 - \frac{r_7}{r} - \frac{r^2}{r_6^2}\right)c^2dt^2 + \frac{dr^2}{\left(1 - \frac{r_7}{r} - \frac{r^2}{r_6^2}\right)} + r^2d\theta^2 - r^2\sin^2\theta\, d\varphi^2,$$

$$ds_4^{(+++-)2} = \left(1 + \frac{r_7}{r} + \frac{r^2}{r_6^2}\right)c^2dt^2 + \frac{dr^2}{\left(1 + \frac{r_7}{r} + \frac{r^2}{r_6^2}\right)} + r^2d\theta^2 - r^2\sin^2\theta\, d\varphi^2,$$

[a][iii] The scope of the d_r^{+}-«quark» (2.9.24)

in the interval [0, ∞]

$$ds_5^{(+++-)2} = c^2dt^2 + dr^2 + r^2d\theta^2 - r^2\sin^2\theta d\varphi^2;$$

And

[b] u_g^{-}-«antiquark»

with signature

$(- + - +)$

which consists of:

[b][i] The outer shell of the u_g^{-}-«antiquark» (2.9.25)

in the interval [r_5, r_6] (Figure 2.9.1)

$$ds_1^{(-+-+)2} = -\left(1 - \frac{r_6}{r} + \frac{r^2}{r_5^2}\right)c^2dt^2 + \frac{dr^2}{\left(1 - \frac{r_6}{r} + \frac{r^2}{r_5^2}\right)} - r^2d\theta^2 + r^2\sin^2\theta\, d\varphi^2,$$

$$ds_2^{(-+-+)2} = -\left(1 + \frac{r_6}{r} - \frac{r^2}{r_5^2}\right)c^2dt^2 + \frac{dr^2}{\left(1 + \frac{r_6}{r} - \frac{r^2}{r_5^2}\right)} - r^2d\theta^2 + r^2\sin^2\theta\, d\varphi^2,$$

$$ds_3^{(-+-+)2} = -\left(1 - \frac{r_6}{r} - \frac{r^2}{r_5^2}\right)c^2dt^2 + \frac{dr^2}{\left(1 - \frac{r_6}{r} - \frac{r^2}{r_5^2}\right)} - r^2d\theta^2 + r^2\sin^2\theta\, d\varphi^2,$$

$$ds_4^{(-+-+)2} = -\left(1 + \frac{r_6}{r} + \frac{r^2}{r_5^2}\right)c^2dt^2 + \frac{dr^2}{\left(1 + \frac{r_6}{r} + \frac{r^2}{r_5^2}\right)} - r^2d\theta^2 + r^2\sin^2\theta\, d\varphi^2,$$

[b][ii] The core of the u_g^{-}-«antiquark» (2.9.26)

in the interval [r_6, r_7] (Figure 2.9.1)

$$ds_1^{(-+-+)2} = -\left(1 - \frac{r_7}{r} + \frac{r^2}{r_6^2}\right)c^2dt^2 + \frac{dr^2}{\left(1 - \frac{r_7}{r} + \frac{r^2}{r_6^2}\right)} - r^2d\theta^2 + r^2\sin^2\theta\, d\varphi^2,$$

$$ds_2^{(-+-+)2} = -\left(1 + \frac{r_7}{r} - \frac{r^2}{r_6^2}\right)c^2dt^2 + \frac{dr^2}{\left(1 + \frac{r_7}{r} - \frac{r^2}{r_6^2}\right)} - r^2d\theta^2 + r^2\sin^2\theta\, d\varphi^2,$$

$$ds_3^{(-+-+)2} = -\left(1 - \frac{r_7}{r} - \frac{r^2}{r_6^2}\right)c^2dt^2 + \frac{dr^2}{\left(1 - \frac{r_7}{r} - \frac{r^2}{r_6^2}\right)} - r^2d\theta^2 + r^2\sin^2\theta\, d\varphi^2,$$

$$ds_4^{(-+-+)2} = -\left(1 + \frac{r_7}{r} + \frac{r^2}{r_6^2}\right)c^2dt^2 + \frac{dr^2}{\left(1 + \frac{r_7}{r} + \frac{r^2}{r_6^2}\right)} - r^2d\theta^2 + r^2\sin^2\theta\, d\varphi^2$$

[b][iii] The scope of the u_g^--«antiquark» (2.9.27)
in the interval [0, ∞]

$$ds_5^{(-+-+)2} = -c^2dt^2 + dr^2 - r^2d\theta^2 + r^2\sin^2\theta d\varphi^2;$$

And

[c] u_b^--«antiquark» (2.9.28)

with signature
(− − + +)

[c][i] The outer shell of the u_b^--«antiquark»
in the interval [r_5, r_6] (Figure 2.9.1):

$$ds_1^{(--++)2} = -\left(1 - \frac{r_6}{r} + \frac{r^2}{r_5^2}\right)c^2dt^2 - \frac{dr^2}{\left(1 - \frac{r_6}{r} + \frac{r^2}{r_5^2}\right)} + r^2d\theta^2 + r^2\sin^2\theta d\varphi^2,$$

$$ds_2^{(--++)2} = -\left(1 + \frac{r_6}{r} - \frac{r^2}{r_5^2}\right)c^2dt^2 - \frac{dr^2}{\left(1 + \frac{r_6}{r} - \frac{r^2}{r_5^2}\right)} + r^2d\theta^2 + r^2\sin^2\theta d\varphi^2,$$

$$ds_3^{(--++)2} = -\left(1 - \frac{r_6}{r} - \frac{r^2}{r_5^2}\right)c^2dt^2 - \frac{dr^2}{\left(1 - \frac{r_6}{r} - \frac{r^2}{r_5^2}\right)} + r^2d\theta^2 + r^2\sin^2\theta d\varphi^2,$$

$$ds_4^{(--++)2} = -\left(1 + \frac{r_6}{r} + \frac{r^2}{r_5^2}\right)c^2dt^2 - \frac{dr^2}{\left(1 + \frac{r_6}{r} + \frac{r^2}{r_5^2}\right)} + r^2d\theta^2 + r^2\sin^2\theta d\varphi^2;$$

[c][ii] The core of the u_b^--«antiquark» (2.9.29)
in the interval [r_6, r_7] (Figure 2.9.1)

$$ds_1^{(--++)2} = -\left(1 - \frac{r_7}{r} + \frac{r^2}{r_6^2}\right)c^2dt^2 - \frac{dr^2}{\left(1 - \frac{r_7}{r} + \frac{r^2}{r_6^2}\right)} + r^2d\theta^2 + r^2\sin^2\theta d\varphi^2,$$

$$ds_2^{(--++)2} = -\left(1 + \frac{r_7}{r} - \frac{r^2}{r_6^2}\right)c^2dt^2 - \frac{dr^2}{\left(1 + \frac{r_7}{r} - \frac{r^2}{r_6^2}\right)} + r^2d\theta^2 + r^2\sin^2\theta d\varphi^2,$$

$$ds_3^{(--++)2} = -\left(1 - \frac{r_7}{r} - \frac{r^2}{r_6^2}\right)c^2dt^2 - \frac{dr^2}{\left(1 - \frac{r_7}{r} - \frac{r^2}{r_6^2}\right)} + r^2d\theta^2 + r^2\sin^2\theta d\varphi^2,$$

$$ds_4^{(--++)2} = -\left(1 + \frac{r_7}{r} + \frac{r^2}{r_6^2}\right)c^2dt^2 - \frac{dr^2}{\left(1 + \frac{r_7}{r} + \frac{r^2}{r_6^2}\right)} + r^2d\theta^2 + \sin^2\theta d\varphi^2;$$

[c][iii] The scope of the u_b^--«antiquark» (2.9.30)

in the interval [0, ∞]:

$$ds_5^{(--++)2} = -c^2dt^2 - dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\varphi^2 .$$

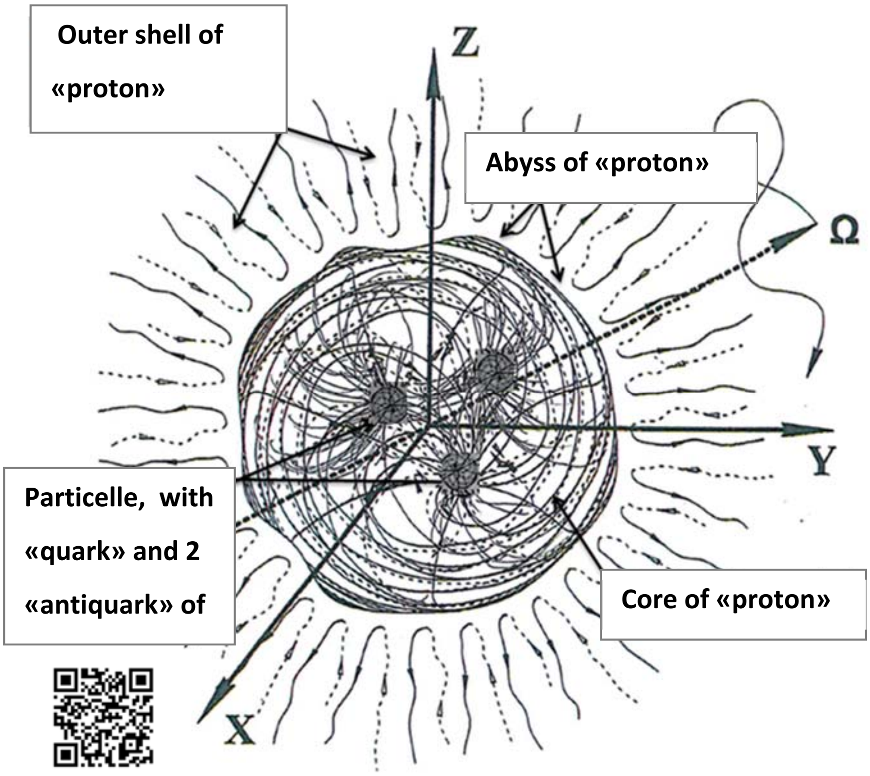


Fig. 2.9.1 A «proton» core essentially consists of the combination of a core with a valence d_r^+-«quark» and two valence u_g^- and u_b^--«antiquarks». Three internal particelles of valence «quarks» are in constant random motion and intertwining with each other

When averaging in homogeneous terms of the metrics (2.9.22) through (2.9.30), we obtain a set of metrics (2.6.32), describing the metric-dynamic state which we have named a «positron». However, it should be expected that the range of the «proton» core, consisting of three «quarks», is greater than the radius of the «positron» core, as the three «quarks» of the core repel one another away from their common center, where we set $r = 0$ (Figure 2.9.1).

The problem of confinement of one «quark» and two «antiquarks» is immediately solved, because each «quark» or «antiquark» is an unstable "convex-concave" state of the vacuum region. Only together, do they form a conditionally concave vacuum state with a stable average, thus forming an «proton» (Figure 2.9.1).

The average set of metrics (2.9.22) through (2.9.30) is a part of the solution of the simplified third Einstein field equation (2.6.21), as well as a set of metrics (2.6.32).

The «quarks» u_g^-, u_b^-, d_r^+ are in chaotic motion with respect to the common center at $r = 0$ and relative to one other (Figure 2.9.1). On the average, they will thus make up an «proton»: $\langle r_g \rangle = r = 0$,

$\langle r_b \rangle = r = 0$, $\langle r_r \rangle = r = 0$. Therefore, we have to use not only the metric-dynamic but also the statistical description of intranuclear processes; a fuller discussion of this may be found in chapters 3 and 4.

The mathematical methods which have been briefly touched upon in § 2.1 to § 2.3 of this Chapter, and developed more fully in [22]. These allow one to retrieve information on a variety of processes and sub-processes that occur within the core, and in its outer shell of the «proton», from the set of metrics (2.9.22) through (2.9.30).

2.10 The «neutron»

In modern nuclear physics, the neutron consists of two *d*-quarks with a charge of $(-1/3)e$ and a *u*-quark with a charge $(2/3)e$ (where e – an electron charge)

$$n = ddu. \tag{2.10.1}$$

As a result of this combination, a neutron is an electrically neutral particle with zero net charge $(-1/3)\,e + (-1/3)\,e + (2/3)\,e = 0$.

In the Algebra of Signatures 3-«quark» particles with zero electric charge does not work! Since there is no additive combination of three of the 16 signatures (2.8.12) leading to a zero signature (0 0 0 0), which means in fact that all sub-contact - antisub-circuit intra-vacuum currents in the outer shell of such a "particle" are completely mutually compensated.

In the framework of the Algebra of Signatures, no metric-dynamic model of a 3-«quark» particle with zero "electric" charge **is available!** This is due to the fact that there is no additive combination of three of the 16 signatures (2.8.12) leading to a zero signature (0 0 0 0), which means in fact that all subcont-antisubcont intra-vacuum currents in the outer shell of such a «particle» sum to zero in combinations of an even number of signatures.

However, the desired result is achieved in the case of the rankings which we have outlined, consisting of four signatures. Therefore, the "electrically" neutral «particle» («neutron») may have the following topology (node) configurations:

$$\begin{array}{ll} i_w^- & (-\ -\ -\ -) \\ d_b^+ & (+\ -\ +\ +) \\ u_r^- & (-\ +\ +\ -) \\ d_g^+ & \underline{(+\ +\ -\ +)} \\ n_1^0 & (0\ \ 0\ \ 0\ \ 0)_+ \end{array} \qquad \begin{array}{ll} i_w^- & (-\ -\ -\ -) \\ d_g^+ & (+\ +\ -\ +) \\ d_r^+ & (+\ +\ +\ -) \\ u_b^- & \underline{(-\ -\ +\ +)} \\ n_2^0 & (0\ \ 0\ \ 0\ \ 0)_+ \end{array} \qquad \begin{array}{ll} i_w^- & (-\ -\ -\ -) \\ d_b^+ & (+\ -\ +\ +) \\ u_g^- & (-\ +\ -\ +) \\ d_r^+ & \underline{(+\ +\ +\ -)} \\ n_3^0 & (0\ \ 0\ \ 0\ \ 0)_+ \end{array} \qquad \begin{array}{ll} i_w^- & (-\ -\ -\ -) \\ u_g^- & (-\ +\ -\ +) \\ d_b^+ & (+\ -\ +\ +) \\ d_r^+ & \underline{(+\ +\ +\ -)} \\ n_4^0 & (0\ \ 0\ \ 0\ \ 0)_+ \end{array}$$

$$\begin{array}{ll} i_w^+ & (+\ +\ +\ +) \\ d_b^- & (-\ +\ -\ -) \\ u_r^+ & (+\ -\ -\ +) \\ d_g^- & \underline{(-\ -\ +\ -)} \\ n_5^0 & (0\ \ 0\ \ 0\ \ 0)_+ \end{array} \qquad \begin{array}{ll} i_w^+ & (+\ +\ +\ +) \\ d_g^- & (-\ -\ +\ -) \\ d_r^- & (-\ -\ -\ +) \\ u_b^+ & \underline{(+\ +\ -\ -)} \\ n_6^0 & (0\ \ 0\ \ 0\ \ 0)_+ \end{array} \qquad \begin{array}{ll} i_w^+ & (+\ +\ +\ +) \\ d_b^- & (-\ +\ -\ -) \\ u_g^+ & (+\ -\ +\ -) \\ d_r^- & \underline{(-\ -\ -\ +)} \\ n_7^0 & (0\ \ 0\ \ 0\ \ 0)_+ \end{array} \qquad \begin{array}{ll} i_w^+ & (+\ +\ +\ +) \\ u_g^+ & (+\ -\ +\ -) \\ d_b^- & (-\ +\ -\ -) \\ d_r^- & \underline{(-\ -\ -\ +)} \\ n_8^0 & (0\ \ 0\ \ 0\ \ 0)_+ \end{array}$$

where

10 metrics are of the form (2.6.22) with signature (+ + + +): a white i_w^+-«quark»; (2.10.2)

10 metrics are of the form (2.6.22) with signature (− − − −): a white i_w^--anti-2-quark. (2.10.3)

White «quarks» are so named because they are almost invisible within the core of the «neutron», since from the point of view of topology, they are a point of (2.8.13) and an anti-point of (2.8.14). Apparently, therefore, their presence in the core of the «neutron» was not detected experimentally, and was not taken into account by the Standard Model.

Thus, under the methods of the Algebra of Signatures, eight possible states of the «neutron» can be represented as:

$$n_1^0 = i_w^- d_b^+ d_g^+ u_r^-, \quad n_2^0 = i_w^- d_r^+ d_g^+ u_b^-, \quad n_3^0 = i_w^- d_r^+ d_b^+ u_g^-, \quad n_4^0 = i_w^- d_r^+ d_b^+ u_g^-, \tag{2.10.4}$$
$$n_5^0 = i_w^+ d_b^- d_g^- u_r^+, \quad n_6^0 = i_w^+ d_g^- d_r^- u_b^+, \quad n_7^0 = i_w^+ d_b^- d_r^- u_g^+, \quad n_8^0 = i_w^+ d_b^- d_r^- u_g^+,$$

similar to the neutron in the Standard Model (2.10.1).

Due to the complicated "intra-core" topological metamorphosis, any additive 4-«quark» combination (2.10.2) can be reconstructed so that the inside of the vacuum formation will consist of an «proton» and an «electron»:

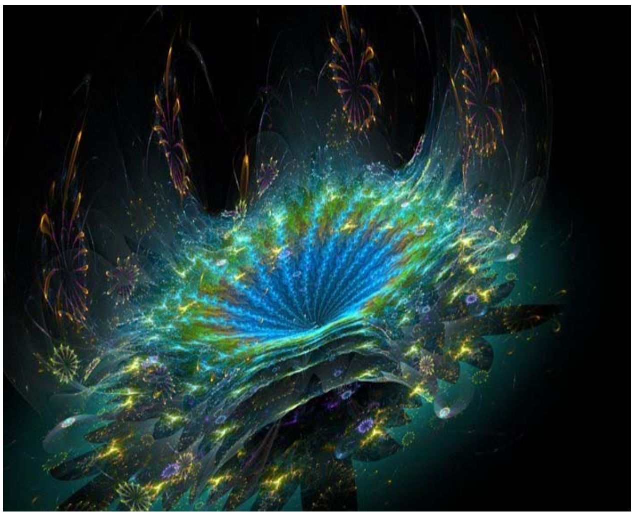
$$\begin{array}{c} (-\ -\ -\ -) \\ (+\ -\ +\ +) \\ (-\ +\ +\ -) \\ \underline{(+\ +\ -\ +)} \\ (0\ 0\ 0\ 0)_+ \end{array} \longrightarrow \begin{array}{c} (+\ -\ +\ +) \\ (-\ +\ +\ -) \\ (-\ +\ -\ +) \\ \underline{(+\ -\ -\ -)} \\ (0\ 0\ 0\ 0)_+ \end{array} \begin{array}{l} \left.\begin{array}{l} \\ \\ \\ \end{array}\right\} \boxed{\text{«proton»}} \\ \leftarrow \boxed{\text{«electron»}} \\ \\ \end{array}$$

(2.10.5)

Apparently, this rebuilding ("unleashing") of a topological node inside the core, the «neutron», leads to the decomposition reaction

$$n \to p^+ + e^- + \nu_e, \tag{2.10.6}$$

where ν_e is an «neutrino» (metric-dynamic models of various «neutrino» are considered in Chapter 7).



2.11 The hydrogen «atom»

Compared with the «neutron» a substantially more stable neutral vacuum formation is the «atom» of hydrogen.

According to astronomical observations, visible matter in the Universe consists of approximately three quarters hydrogen and approximately a quarter helium, with the other chemical elements accounting for only around two percent.

A neutral atom of deuterium is composed of one «proton», one «neutron» and one «electron. As part of the Algebra" of signatures, it turns out that the «atom» of deuterium is composed of an «proton», an neutron and an «electron». The rank (topological) equivalent nodal configuration of such a region of the vacuum is as follows:

$$\begin{array}{cccc}
\begin{array}{c}\text{«proton»}\\ \\ \\ + \\ \text{«neutron»}\\ + \\ \\ \text{«electron»}\\ = \end{array}
&
\begin{array}{l}
\left[\begin{array}{cccc}(+&+&+&-)\\(-&+&-&+)\\(-&-&+&+)\end{array}\right.\\
\left[\begin{array}{cccc}(-&-&-&-)\\(+&-&+&+)\\(-&+&+&-)\\(+&+&-&+)\end{array}\right.\\
\left\{\begin{array}{cccc}\underline{(+}&\underline{-}&\underline{-}&\underline{-)}\end{array}\right.\\
{}^{1}\mathrm{H}(0\ \ 0\ \ 0\ \ 0)_{+}
\end{array}
&
\text{or}
&
\begin{array}{l}
\begin{array}{cccc}(+&+&-&+)\\(-&-&+&+)\\(-&+&+&-)\\(+&+&+&+)\\(+&-&+&-)\\(-&+&-&-)\\(-&-&-&+)\\ \underline{(+}&\underline{-}&\underline{-}&\underline{-)}\end{array}\\
{}^{1}\mathrm{H}(0\ \ 0\ \ 0\ \ 0)_{+}
\end{array}
\ \text{or}\ \ldots
\end{array}
\qquad (2.11.1)$$

Many combinations of signatures similar to (2.11.1) can be made, which reflects the possibilities of "color" combinatorics of intra-core metamorphoses. But the topological configuration of this "node" always remains the same: three u-«quarks», three d-«quarks», one i-«quark» and one e-«quark». We agree to denote such a topological "node" as follows:

$$^{1}H = 3u3die\,, \qquad (2.11.2)$$

Taking into account the topological properties of the metric with the appropriate signatures (2.8.13) through (2.8.16), we find that the "node" consists of three twisted "torahs", four oval surfaces and a "point".

Similarly, all the known chemical elements of the Mendeleev's periodic table could be constructed, or following up on our previous image, braided, whereby the average size of their nuclei r_n would depend on the number of «quarks» A forming the "topology nodes":

$$r_n \approx ½A^{1/3} r_6 \approx ½\, A^{1/3} \cdot 10^{-13}\text{cm}\,.$$

It is tempting to postulate that these discrete radii in stable vacuum states form a Fibonacci or other Lucas sequence. To follow up on this idea, a task which we shall not attempt here, an appropriate starting point would be to apply equation (2.7.8) with $r_k = r_6$.

2.12 «Fermions» in the Algebra of Signatures

Having a set out of 16 colored «quarks» (2.9.15) through (2.9.18) and (2.10.3) (as summarized in Table 2.12.1) and understanding their topological features, all fermions (mesons and baryons) from the Standard Model can be braided.

Table 2.12.1

«Quarks»		«Antiquarks»	
10 metrics type (2.6.22) or (2.12.1) with signature:	«quark»	10 metrics type (2.6.22) or (2.12.1) with signature:	«antiquark»
(+ – – –)	e^+-«quark», or «electron»	(– + + +)	e^--«antiquark», or «positron»
(+ + + –) (+ + – +) (+ – + +)	d_r^+-«quark» d_g^+-«quark» d_b^+-«quark»	(– – – +) (– – + –) (– + – –)	d_r^--«antiquark» d_g^--«antiquark» d_b^--«antiquark»
(+ – – +) (+ – + –) (+ + – –)	u_r^+-«quark» u_g^+-«quark» u_b^+-«quark»	(– + + –) (– + – +) (– – + +)	u_r^--«antiquark» u_g^--«antiquark» u_b^--«antiquark»
(+ + + +)	i_w^+-«quark» (“invisibles”)	(– – – –)	i_w^--«antiquark» (“anti-invisibles”)

where, for example,

u_k^{-}-«antiquark» (2.12.1)

with signature is $(-++-)$
composed of:

The outer shell of the u_k^{-}-«antiquark» (2.12.2)

in the interval $[r_5, r_6]$ (Figure 2.9.1)

$$ds_1^{(-)2} = -\left(1 - \frac{r_6}{r} + \frac{r^2}{r_5^2}\right)c^2dt^2 + \frac{dr^2}{\left(1 - \frac{r_6}{r} + \frac{r^2}{r_5^2}\right)} + r^2d\theta^2 - r^2\sin^2\theta d\varphi^2,$$

$$ds_2^{(-)2} = -\left(1 + \frac{r_6}{r} - \frac{r^2}{r_5^2}\right)c^2dt^2 + \frac{dr^2}{\left(1 + \frac{r_6}{r} - \frac{r^2}{r_5^2}\right)} + r^2d\theta^2 - r^2\sin^2\theta d\varphi^2,$$

$$ds_3^{(-)2} = -\left(1 - \frac{r_6}{r} - \frac{r^2}{r_5^2}\right)c^2dt^2 + \frac{dr^2}{\left(1 - \frac{r_6}{r} - \frac{r^2}{r_5^2}\right)} + r^2d\theta^2 - r^2\sin^2\theta d\varphi^2,$$

$$ds_4^{(-)2} = -\left(1 + \frac{r_6}{r} + \frac{r^2}{r_5^2}\right)c^2dt^2 + \frac{dr^2}{\left(1 + \frac{r_6}{r} + \frac{r^2}{r_5^2}\right)} + r^2d\theta^2 - r^2\sin^2\theta d\varphi^2;$$

The core of the u_k^{-}-«antiquark» (2.12.3)

in the interval $[r_6, r_7]$ (Figure 2.9.1)

$$ds_1^{(-)2} = -\left(1 - \frac{r_7}{r} + \frac{r^2}{r_6^2}\right)c^2dt^2 + \frac{dr^2}{\left(1 - \frac{r_7}{r} + \frac{r^2}{r_6^2}\right)} + r^2d\theta^2 - r^2\sin^2\theta d\varphi^2,$$

$$ds_2^{(-)2} = -\left(1 + \frac{r_7}{r} - \frac{r^2}{r_6^2}\right)c^2dt^2 + \frac{dr^2}{\left(1 + \frac{r_7}{r} - \frac{r^2}{r_6^2}\right)} + r^2d\theta^2 - r^2\sin^2\theta d\varphi^2,$$

$$ds_3^{(-)2} = -\left(1 - \frac{r_7}{r} - \frac{r^2}{r_6^2}\right)c^2dt^2 + \frac{dr^2}{\left(1 - \frac{r_7}{r} - \frac{r^2}{r_6^2}\right)} + r^2d\theta^2 - r^2\sin^2\theta d\varphi^2,$$

$$ds_4^{(-)2} = -\left(1 + \frac{r_7}{r} + \frac{r^2}{r_6^2}\right)c^2dt^2 + \frac{dr^2}{\left(1 + \frac{r_7}{r} + \frac{r^2}{r_6^2}\right)} + r^2d\theta^2 - \sin^2\theta d\varphi^2,$$

The scope of the u_k^{-}-«antiquark» (2.12.4)

in the interval $[0, \infty]$

$$ds_5^{(-)2} = -c^2dt^2 + dr^2 + r^2d\theta^2 - r^2\sin^2\theta d\varphi^2.$$

In quantum chromodynamics, mesons are composed of a quark and an antiquark, and are given by

$$M = q^- q^+ = q_\alpha^- q_\alpha^+ = \frac{1}{\sqrt{3}}\left(q_{г}^- q_{г}^+ + q_{к}^- q_{к}^+ + q_{з}^- q_{з}^+\right), \qquad (2.12.5)$$

where q_α^+ ($\alpha = b, g, r$) is a quark (or antiquark) color triplet, and q_α^- is an antiquark color triplet.

Baryons composed of 3 quarks, and are given by

$$B = \frac{1}{\sqrt{6}} q_\alpha q_\beta q_\gamma \varepsilon_{\alpha\beta\gamma} , \qquad (2.12.6)$$

where $\varepsilon_{\alpha\beta\gamma}$ are completely antisymmetric tensor.

«Mesons» and «baryons» are formed in the same way in the Algebra of Signatures. Consider a specific example: three types of pi-mesons subject to strong interactions have the quark structure:

$$\pi^+ = u^- d^+, \qquad \pi^0 = \frac{1}{\sqrt{2}}\left(u^- u^+ - d^+ d^-\right), \qquad \pi^- = u^+ d^-. \qquad (2.12.7)$$

In the Algebra of signatures, such as the meson $\pi^+ = u^- d^+$ is represented as

$$\begin{array}{lll}
d_r^+ \, (+ + + -) & d_g^+ \, (+ + - +) & d_b^+ \, (+ - + +) \\
\underline{u_g^- \, (- + - +)} & \underline{u_b^- \, (- - + +)} & \underline{u_r^- \, (- + + -)} \\
\pi_1^+ \, (0\ 2{+}\ 0\ 0)_+ & \pi_2^+ \, (0\ 0\ 0\ 2{+})_+ & \pi_3^+ \, (0\ 0\ 2{+}\ 0)_+
\end{array} \qquad (2.12.8)$$

for which each signature corresponds to the set of ten metrics of the type (2.12.1).

Even from within these ranks it is seen that such a convex-concave vacuum formation cannot be stable. They can arise from this topological configuration, but in this way, they instantly disappear, blur together or collapse to nodes resulting from the intertwining of the inside vacuum currents in the curved region of the vacuum.

In turn, the «quark» structure

$$\pi^0 = \frac{1}{\sqrt{2}}\left(u^- u^+ - d^+ d^-\right) \qquad (2.12.9)$$

can have the following signature (topological) analogues:

$$\begin{array}{llllll}
 & u_r^+ \, (+ - - +) & & u_g^+ \, (+ - + -) & & u_b^+ \, (+ + - -) \\
 & u_g^- \, (- + - +)_+ & & u_b^- \, (- - + +)_+ & & u_r^- \, (- + + -)_+ \\
- & & - & & - & \\
 & d_r^+ \, (+ + + -) & & d_g^+ \, (+ + - +) & & d_b^+ \, (+ - + +) \\
 & \underline{d_g^- \, (- - + -)}_+ & & \underline{d_b^- \, (- + - -)}_+ & & \underline{d_r^- \, (- - - +)}_+ \\
 & \pi_1^0 \, (0\ 0\ 0\ 0) & & \pi_2^0 \, (0\ 0\ 0\ 0) & & \pi_3^0 \, (0\ 0\ 0\ 0)
\end{array}$$

(2.12.10)

Similarly, under the Algebra of Signature all known mesons and baryons from the Standard Model can be braided.

The Algebra of Signatures differs from the Standard Model only in the presence of its other "invisible": i_b^+-«quark» and i_b^--«antiquark».

2.13 «Bosons» in the Algebra of Signatures

The local part of the flat outer side of the vacuum region is described by the metric (2.8.1)

$$ds^{(-)2} = c^2dt^2 - dx^2 - dy^2 - dz^2 = \eta_{ij}^{(-)}dx^i dx^j \text{ with the signature } (+\,-\,-\,-), \qquad (2.13.1)$$

where

$$\eta_{ij}^{(-)} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \qquad (2.13.2)$$

and the same lengths of the inside of the vacuum region is described by the metric (2.8.2)

$$ds^{(+)2} = -c^2dt^2 + dx^2 + dy^2 + dz^2 = \eta_{ij}^{(+)}dx^i dx^j \text{ with signature } (-\,+\,+\,+)$$

where

$$\eta_{ij}^{(+)} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \qquad (2.13.3)$$

As part of the Algebra of Signatures, weak perturbations of a two-way vacuum over a given 2-braid (averaged metric) take the form

$$½(ds^{(-)2} + ds^{(+)2}) = ½(\eta_{ij}^{(-)} + h_{ij}^{(-)} + \eta_{ij}^{(+)} - h_{ij}^{(+)})\, dx^i dx^j = ½(h_{ij}^{(-)} - h_{ij}^{(+)})\, dx^i dx^j, \qquad (2.13.4)$$

where $h_{ij}^{(-)}$ and $h_{ij}^{(+)}$ are related components of the tensors defining slight bilateral deviations from the state of the original uncurved vacuum region.

We assume a fixed reference system in a fashion similar to the fixing of the electromagnetic vector potential in the Lorentz gauge condition in electrodynamics [34]. We further impose additional conditions on $h_{ij}^{(-)}$ and $h_{ij}^{(+)}$, so that the first vacuum Einstein equation (2.1.6) is reduced to the wave equation

$$R_{ij} \approx \left(\nabla - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\frac{1}{2}(h_{ij}^{(-)} - h_{ij}^{(+)}) = 0. \qquad (2.13.5)$$

In a small area of the vacuum, the wave disturbance can be regarded as a plane wave. If the direction of wave propagation is represented along the x-axis, a suitable choice of the reference system will make the components $h_{ij}^{(-)}$ and $h_{ij}^{(+)}$ vanish, as well as the components

$$h_{22}^{(-)} = -h_{33}^{(-)} \equiv h_+^{(-)} \quad \text{and} \quad h_{32}^{(-)} = h_{23}^{(-)} \equiv h_\times^{(-)}. \tag{2.13.6}$$

$$h_{22}^{(+)} = -h_{33}^{(+)} \equiv h_+^{(+)} \quad \text{and} \quad h_{32}^{(+)} = h_{23}^{(+)} \equiv h_\times^{(+)}.$$

Such a wave disturbance is a quadrupolar transverse wave. The polarization of this wave in the *u-z* plane is defined by the following tensor of the second rank:

$$h_{ab}^{(-)} = \begin{pmatrix} h_+^{(-)} & h_\times^{(-)} \\ h_\times^{(-)} & -h_+^{(-)} \end{pmatrix} = 0, \qquad h_{ab}^{(+)} = \begin{pmatrix} h_+^{(+)} & h_\times^{(+)} \\ h_\times^{(+)} & -h_+^{(+)} \end{pmatrix} = 0, \qquad a, b = 2, 3. \tag{2.13.7}$$

The separate components, $h_+^{(-)}$ and $h_\times^{(-)}$, $h_+^{(+)}$ and $h_\times^{(+)}$, describe two independent polarization planes of the quadrupolar wave disturbances which differ from each other by a rotation through an angle of $\pi/4$.

The average second-rank tensor

$$h_{ab}^{(\pm)} = \frac{1}{2}\begin{pmatrix} h_+^{(-)} - h_+^{(+)} & h_\times^{(-)} - h_\times^{(+)} \\ h_\times^{(-)} - h_\times^{(+)} & -h_+^{(-)} + h_+^{(+)} \end{pmatrix} = 0, \tag{2.13.8}$$

can describe, under certain phase relationships, not only the quadrupolar but also the dipolar, including linear, elliptical and circular polarization wave disturbances of a two-sided extension.

Thus, the first Einstein field equation (2.1.6) is linearized for small perturbations of the metric, i.e., it becomes the wave (2.13.5), and allows the distribution of different types of wave disturbances on the two-sided vacuum region.

The problem of propagation of wave disturbances throughout the vacuum can be considered in a different way. We start with the metric (2.13.1)

$$ds^{(-)2} = c^2dt^2 - dx^2 - dy^2 - dz^2 = 0 \quad \text{with the signature } (+ - - -). \tag{2.13.9}$$

This determines not only the metric-dynamic properties of the flat outer side of the vacuum region, but also the spread of the light beam in a vacuum at a forward speed of $cdt = (dx^2 + dy^2 + dz^2)^{1/2}$.

In this metric (2.13.3)

$$ds^{(+)2} = -c^2dt^2 + dx^2 + dy^2 + dz^2 = 0 \quad \text{with signature } (- + + +). \tag{2.13.10}$$

determines not only the metric-dynamic properties of the flat inner side of the vacuum region, but also the spread of the light beam in a vacuum at a speed from the opposite direction

$$-cdt = -(dx^2 + dy^2 + dz^2)^{1/2}.$$

Recall that the quadratic form (2.13.9) and (2.13.10) can be represented as a product of linear (affine) forms (2.1.37) and (2.1.38)

$$ds^{(-)2} = c^2dt^2 - dx^2 - dy^2 - dz^2 = cdt'cdt'' - dx'dx'' - dy'dy'' - dz'dz'', \qquad (2.13.11)$$

$$ds^{(+)2} = -c^2dt^2 + dx^2 + dy^2 + dz^2 = -cdt'cdt'' + dx'dx'' + dy'dy'' + dz'dz'', \qquad (2.13.12)$$

where, according to (2.1.39) through (2.1.42):

$ds^{(-)\prime} = cdt' - dx' - dy' - dz'$: "Cover" of the outer side of the vacuum; (2.13.13)

$ds^{(-)\prime\prime} = cdt'' - dx'' - dy'' - dz''$: "Inside" of the outer side of the vacuum; (2.13.14)

$ds^{(+)\prime} = -cdt' + dx' + dy' + dz'$: "Cover" of the inner side of the vacuum; (2.13.15)

$ds^{(+)\prime\prime} = -cdt'' + dx'' + dy'' + dz''$: "Inside" of the inner side of the vacuum. (2.13.16)

Since the segments from (2.13.13) through (2.13.16) are perpendicular to each other:

$$ds^{(-)\prime} \perp ds^{(-)\prime\prime} \perp ds^{(+)\prime} \perp ds^{(+)\prime\prime},$$

the language of quaternions is the most effective form to handle them.

In that case, instead of the linear form (2.13.13), we use quaternion

$$z = -x_0 + ix_1 + jx_2 + kx_3, \quad stignature\ \{-+++\} \qquad (2.13.17)$$

and instead of (2.13.15) , the complex conjugate quaternion

$$z^* = x_0 - xi_3 - jx_2 - kx_1, \quad stignature\ \{+---\} \qquad (2.13.18)$$

In general, the Algebra of Signatures admits the existence of 16 types of "color" quaternions with all possible stignatures:

$z_1 = x_0 + ix_1 + jx_2 + kx_3$	$\{++++\}$	$\{----\}$	$z_9 = -x_0 - ix_1 - jx_2 - kx_3$
$z_2 = -x_0 - ix_1 - jx_2 + kx_3$	$\{---+\}$	$\{+++-\}$	$z_{10} = x_0 + ix_1 + jx_2 - kx_3$
$z_3 = x_0 - ix_1 - jx_2 + kx_3$	$\{+--+\}$	$\{-++-\}$	$z_{11} = -x_0 + ix_1 + jx_2 - kx_3$
$z_4 = -x_0 - ix_1 + jx_2 - kx_3$	$\{--+-\}$	$\{++-+\}$	$z_{12} = x_0 + ix_1 - jx_2 + kx_3$
$z_5 = x_0 + ix_1 - jx_2 - kx_3$	$\{++--\}$	$\{--++\}$	$z_{13} = -x_0 - ix_1 + jx_2 + kx_3$
$z_6 = -x_0 + ix_1 - jx_2 - kx_3$	$\{-+--\}$	$\{+-++\}$	$z_{14} = x_0 - ix_1 + jx_2 + kx_3$
$z_7 = x_0 - ix_1 + jx_2 - kx_3$	$\{+-+-\}$	$\{-+-+\}$	$z_{15} = -x_0 + ix_1 - jx_2 + kx_3$
$z_8 = -x_0 + ix_1 + jx_2 + kx_3$	$\{-+++\}$	$\{+---\}$	$z_{16} = x_0 - ix_1 - jx_2 - kx_3$

(2.13.19)

By a straightforward calculation, it is easy to see that the sum of all 16 types of "color" quaternions (2.13.19) is equal to zero

$$z_1 + z_2 + z_3 + z_4 + z_5 + z_6 + z_7 + z_8 + z_9 + z_{10} + z_{11} + z_{12} + z_{13} + z_{14} + z_{15} + z_{16} = 0, \qquad (2.13.20)$$

so that we can consider that the vacuum itself satisfies the “vacuum condition”.

Equivalent stignatures from (2.13.20) take on the form:

$$\begin{aligned} &\{++++\} + \{---+\} + \{+--+\} + \{++--\} + \\ &+ \{+-+-\} + \{-+--\} + \{+-+-\} + \{-+++\} + \\ &+ \{----\} + \{+++-\} + \{-++-\} + \{++-+\} + \\ &+ \{--++\} + \{+-++\} + \{-+-+\} + \{+---\} = \{0000\}. \end{aligned} \qquad (2.13.21)$$

stignatures form a structure similar to the signature structure and antisymmetric matrix referred to earlier in this paper

$$*-stignatures=\begin{matrix}\{++++\} & \{+++-\} & \{-++-\} & \{++-+\}\\ \{---+\} & \{-+++\} & \{--++\} & \{-+-+\}\\ \{+--+\} & \{++--\} & \{+---\} & \{+-++\}\\ \{--+-\} & \{+-+-\} & \{-+--\} & \{----\}\end{matrix} \qquad (2.13.22)$$

A more detailed analysis of the 16 aggregate stignatures and the "colored" quaternions is given in Chapter 1.

13.1 The «photon» and «antiphoton»

Because, for example, the linear forms (2.13.13) and (2.13.14) are mutually perpendicular in relation to each other; the harmonic perturbation propagating along the total metric length (i.e., on the outer side of the vacuum) can be represented as:

$$\cos\{(2\pi/\lambda)(ct-x-y-z)\} + i\sin\{(2\pi/\lambda)(ct-x-y-z)\} = \exp\{i\,(2\pi/\lambda)(ct-x-y-z)\} = \exp\{i(\omega t-\mathbf{k}\cdot\mathbf{r})\}. \quad (2.13.23)$$

We call such a harmonic disturbance of the metric an «photon» having a metric with stignature $\{+---\}$.

Similarly, for mutually perpendicular linear forms (2.13.15) and (2.13.16) we have a harmonic perturbation of the inner side of the vacuum extension:

$$\cos\{(2\pi/\lambda)(-ct+x+y+z)\} + i\sin\{(2\pi/\lambda)(-ct+x+y+z)\} = \exp\{i\,(2\pi/\lambda)(-ct+x+y+z)\} = \exp-\{i(\omega t-\mathbf{k}\cdot\mathbf{r})\}. \quad (2.13.24)$$

which we call «antiphoton» with stignature $\{-+++\}$ because it extends in the opposite direction with respect to the «photon». (This is not to be confused with the antimatter particle of the photon, which is of course the photon itself).

13.2 The $W^{\pm}$-«bosons»

Similar constructions show that six signature ranks:

$$\begin{matrix}\{---+\} & \{--+-\} & \{-+--\}\\ \{+-+-\} & \{++--\} & \{+--+\}\\ \underline{\{++--\}} & \underline{\{+--+\}} & \underline{\{+-+-\}}\\ \{+---\}_+ & \{+---\}_+ & \{+---\}_+\end{matrix} \qquad (2.13.25)$$

$$\begin{matrix}\{+++-\} & \{++-+\} & \{+-++\}\\ \{-+-+\} & \{--++\} & \{-++-\}\\ \underline{\{--++\}} & \underline{\{-++-\}} & \underline{\{-+-+\}}\\ \{-+++\}_+ & \{-+++\}_+ & \{-+++\}_+\end{matrix}$$

correspond to three colored states of the W^{+}-«boson»

$$\begin{array}{ll} \exp\,\{i\,2\pi/\lambda\,(-ct-x-y+z)\}\times & \{-\ -\ -\ +\} \\ \times\exp\,\{j\,2\pi/\lambda\,(\ \ ct-x+y-z)\}\times & \{+\ -\ +\ -\} \\ \times\exp\,\{k\,2\pi/\lambda\,(\ \ ct+x-y-z)\} & \underline{\{+\ +\ -\ -\}} \\ & \{+\ -\ -\ -\}_{+} \end{array}$$

$$\begin{array}{ll} \exp\,\{i\,2\pi/\lambda\,(-ct-x+y-z)\}\times & \{-\ -\ +\ -\} \\ \times\exp\,\{j\,2\pi/\lambda\,(\ \ ct+x-y-z)\}\times & \{+\ +\ -\ -\} \\ \times\exp\,\{k\,2\pi/\lambda\,(\ \ ct-x-y+z)\} & \underline{\{+\ -\ -\ +\}} \\ & \{+\ -\ -\ -\}_{+} \end{array} \qquad (2.13.26)$$

$$\begin{array}{ll} \exp\,\{i\,2\pi/\lambda\,(-ct+x-y-z)\}\times & \{-\ +\ -\ -\} \\ \times\exp\,\{j\,2\pi/\lambda\,(\ \ ct-x-y+z)\}\times & \{+\ -\ -\ +\} \\ \times\exp\,\{k\,2\pi/\lambda\,(\ \ ct-x\ +\mathrm{y}\ -z)\} & \underline{\{+\ -\ +\ -\}} \\ & \{+\ -\ -\ -\}_{+} \end{array}$$

and three colored states of the W^{-}-«boson»

$$\begin{array}{ll} \exp\,\{i\,2\pi/\lambda\,(\ \ ct+x+y-z)\}\times & \{+\ +\ +\ -\} \\ \times\exp\,\{j2\pi/\lambda\,(-ct+x-y+z)\}\times & \{-\ +\ -\ +\} \\ \times\exp\,\{k\,2\pi/\lambda\,(-ct-x+y+z)\} & \underline{\{-\ -\ +\ +\}} \\ & \{-\ +\ +\ +\}_{+} \end{array}$$

$$\begin{array}{ll} \exp\,\{i\,2\pi/\lambda\,(\ \ ct+x-y+z)\}\times & \{+\ +\ -\ +\} \\ \times\exp\,\{j\,2\pi/\lambda\,(-ct-x+y+z)\}\times & \{-\ -\ +\ +\} \\ \times\exp\,\{k\,2\pi/\lambda\,(-ct+x+y-z)\} & \underline{\{-\ +\ +\ -\}} \\ & \{-\ +\ +\ +\}_{+} \end{array} \qquad (2.13.27)$$

$$\begin{array}{ll} \exp\,\{i\,2\pi/\lambda\,(\ \ ct-x+y+z)\}\times & \{+\ -\ +\ +\} \\ \times\exp\,\{j\,2\pi/\lambda\,(-ct+x+y-z)\}\times & \{-\ +\ +\ -\} \\ \times\exp\,\{k\,2\pi/\lambda\,(-ct+x-y+z)\} & \underline{\{-\ +\ -\ +\}} \\ & \{-\ +\ +\ +\}_{+}\,, \end{array}$$

where i, j, k are the imaginary units forming an anticommutative algebra:

$$i^{2}=j^{2}=k^{2}=ijk=-1 \quad \text{and} \quad ij+ji=0. \qquad (2.13.28)$$

2.13.3 The Z^{0}-«bosons»

The six signature ranks

$$\begin{array}{ccc} \begin{array}{c} \{-\ -\ -\ -\} \\ \{+\ -\ +\ +\} \\ \{-\ +\ +\ -\} \\ \underline{\{+\ +\ -\ +\}} \\ \{0\ 0\ 0\ 0\}_{+} \end{array} & \begin{array}{c} \{-\ -\ -\ -\} \\ \{+\ +\ -\ +\} \\ \{+\ +\ +\ -\} \\ \underline{\{-\ -\ +\ +\}} \\ \{0\ 0\ 0\ 0\}_{+} \end{array} & \begin{array}{c} \{-\ -\ -\ -\} \\ \{+\ -\ +\ +\} \\ \{-\ +\ -\ +\} \\ \underline{\{+\ +\ +\ -\}} \\ \{0\ 0\ 0\ 0\}_{+} \end{array} \\ \\ \begin{array}{c} \{+\ +\ +\ +\} \\ \{-\ +\ -\ -\} \\ \{+\ -\ -\ +\} \\ \underline{\{-\ -\ +\ -\}} \\ \{0\ 0\ 0\ 0)_{+} \end{array} & \begin{array}{c} \{+\ +\ +\ +\} \\ \{-\ -\ +\ -\} \\ \{-\ -\ -\ +\} \\ \underline{\{+\ +\ -\ -\}} \\ \{0\ 0\ 0\ 0\}_{+} \end{array} & \begin{array}{c} \{+\ +\ +\ +\} \\ \{-\ +\ -\ -\} \\ \{+\ -\ +\ -\} \\ \underline{\{-\ -\ -\ +\}} \\ \{0\ 0\ 0\ 0\}_{+} \end{array} \end{array}$$

(2.13.29)

correspond to the six color states of the Z^0-«boson»

$$\begin{array}{ll}
\exp\{\;2\pi/\lambda\,(-ct-x-y-z)\}\times & \{-\;-\;-\;-\} \\
\times\exp\{i\,2\pi/\lambda\,(\;\;ct-x+y+z)\}\times & \{+\;-\;+\;+\} \\
\times\exp\{j\,2\pi/\lambda\,(-ct+x+y-z)\}\times & \{-\;+\;+\;-\} \\
\times\exp\{k\,2\pi/\lambda\,(\;\;ct+x-y+z)\} & \underline{\{+\;+\;-\;+\}} \\
 & \{0\;\;0\;\;0\;\;0\}_{+}
\end{array}$$

$$\begin{array}{ll}
\exp\{\;2\pi/\lambda\,(-ct-x-y-z)\}\times & \{-\;-\;-\;-\} \\
\times\exp\{i\,2\pi/\lambda\,(\;\;ct+x-y+z)\}\times & \{+\;+\;-\;+\} \\
\times\exp\{j\,2\pi/\lambda\,(\;\;ct+x+y-z)\}\times & \{+\;+\;+\;-\} \\
\times\exp\{k\,2\pi/\lambda\,(-ct-x+y+z)\} & \underline{\{-\;-\;+\;+\}} \\
 & \{0\;\;0\;\;0\;\;0\}_{+}
\end{array}$$

$$\begin{array}{ll}
\exp\{\;2\pi/\lambda\,(-ct-x-y-z)\}\times & \{-\;-\;-\;-\} \\
\times\exp\{i\,2\pi/\lambda\,(\;\;ct-x+y+z)\}\times & \{+\;-\;+\;+\} \\
\times\exp\{j\,2\pi/\lambda\,(-ct+x-y+z)\}\times & \{-\;+\;-\;+\} \\
\times\exp\{k\,2\pi/\lambda\,(\;\;ct+x+y-z)\} & \underline{\{+\;+\;+\;-\}} \\
 & \{0\;\;0\;\;0\;\;0\}_{+}
\end{array}$$

$$\begin{array}{ll}
\exp\{\;2\pi/\lambda\,(\;\;ct+x+y+z)\}\times & \{+\;+\;+\;+\} \\
\times\exp\{i\,2\pi/\lambda\,(-ct+x-y-z)\}\times & \{-\;+\;-\;-\} \\
\times\exp\{j\,2\pi/\lambda\,(\;\;ct-x-y+z)\}\times & \{+\;-\;-\;+\} \\
\times\exp\{k\,2\pi/\lambda\,(-ct-x+y-z)\} & \underline{\{-\;-\;+\;-\}} \\
 & \{0\;\;0\;\;0\;\;0\}_{+}
\end{array}$$

$$\begin{array}{ll}
\exp\{\;2\pi/\lambda\,(\;\;ct+x+y+z)\}\times & \{+\;+\;+\;+\} \\
\times\exp\{i\,2\pi/\lambda\,(-ct-x+y-z)\}\times & \{-\;-\;+\;-\} \\
\times\exp\{j\,2\pi/\lambda\,(-ct-x-y+z)\}\times & \{-\;-\;-\;+\} \\
\times\exp\{k\,2\pi/\lambda\,(\;\;ct+x-y-z)\} & \underline{\{+\;+\;-\;-\}} \\
 & \{0\;\;0\;\;0\;\;0\}_{+}
\end{array}$$

$$\begin{array}{ll}
\exp\{\;2\pi/\lambda\,(\;\;ct+x+y+z)\}\times & \{+\;+\;+\;+\} \\
\times\exp\{i\,2\pi/\lambda\,(-ct+x-y-z)\}\times & \{-\;+\;-\;-\} \\
\times\exp\{j\,2\pi/\lambda\,(\;\;ct-x+y-z)\}\times & \{+\;-\;+\;-\} \\
\times\exp\{k\,2\pi/\lambda\,(-ct-x-y+z)\} & \underline{\{-\;-\;-\;+\}} \\
 & \{0\;\;0\;\;0\;\;0\}_{+}
\end{array}$$

(2.13.30)

2.13.4 The «graviton»

In the Algebra of Signatures, another «boson» appears, namely, the «graviton».

$$
\begin{array}{rl}
\exp\{\zeta_1 2\pi/\lambda\,(\ \ ct+x+y+z)\} & \{+\ \ +\ \ +\ \ +\} \\
\times\exp\{\zeta_3\, 2\pi/\lambda\,(\ \ ct-x-y+z)\}\times & \{-\ \ -\ \ -\ \ +\} \\
\times\exp\{\zeta_4 2\pi/\lambda\,(-ct-x+y-z)\}\times & \{+\ \ -\ \ -\ \ +\} \\
\times\exp\{\zeta_5\, 2\pi/\lambda\,(\ \ ct+x-y-z)\}\times & \{-\ \ -\ \ +\ \ -\} \\
\times\exp\{\zeta_6 2\pi/\lambda\,(-ct+x-y-z)\}\times & \{+\ \ +\ \ -\ \ -\} \\
\times\exp\{\zeta_7\, 2\pi/\lambda\,(\ \ ct-x+y-z)\}\times & \{-\ \ +\ \ -\ \ -\} \\
\times\exp\{\zeta_8\, 2\pi/\lambda\,(-ct+x+y+z)\}\times & \{+\ \ -\ \ +\ \ -\} \\
\times\exp\{\zeta_1\, 2\pi/\lambda\,(-ct-x-y-z)\}\times & \{-\ \ +\ \ +\ \ +\} \\
\times\exp\{\zeta_2 2\pi/\lambda\,(\ \ ct+x+y-z)\}\times & \{-\ \ -\ \ -\ \ -\} \\
\times\exp\{\zeta_3 2\pi/\lambda\,(-ct+x+y-z)\}\times & \{+\ \ +\ \ +\ \ -\} \\
\times\exp\{\zeta_4 2\pi/\lambda\,(\ \ ct+x-y+z)\}\times & \{-\ \ +\ \ +\ \ -\} \\
\times\exp(\zeta_5\, 2\pi/\lambda\,(-ct-x+y+z)\}\times & \{+\ \ +\ \ -\ \ +\} \\
\times\exp\{\zeta_6\, 2\pi/\lambda\,(\ \ ct-x+y+z)\}\times & \{-\ \ -\ \ +\ \ +\} \\
\times\exp\{\zeta_7 2\pi/\lambda\,(-ct+x-y+z)\}\times & \{+\ \ -\ \ +\ \ +\} \\
\times\exp\{\zeta_8\, 2\pi/\lambda\,(\ \ ct-x-y-z)\} & \{-\ \ +\ \ -\ \ +\} \\
 & \underline{\{+\ \ -\ \ -\ \ -\}} \\
 & \{0\ \ 0\ \ 0\ \ 0\}_+
\end{array}
\tag{2.13.31}
$$

whereby the ζ_m entities satisfy the anticommutative relations of a Clifford algebra.

$$\zeta_m\zeta_k+\zeta_k\zeta_m=0 \text{ for } m\neq k,\ \zeta_m\zeta_m=1, \qquad \text{or} \qquad \zeta_m\zeta_k+\zeta_k\zeta_m=2\delta_{km}, \tag{2.13.32}$$

where δ_{km} is the Kronecker delta ($\delta_{km}=0$ for $m\neq k$ and $\delta_{km}=1$ for $m=k$). One way to define objects and ζ_m entities and the Kronecker delta δ_{km} is presented below:

$$
\zeta_1=\begin{pmatrix}
1&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0\\
0&0&0&1&0&0&0&0\\
0&0&0&0&1&0&0&0\\
0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&1
\end{pmatrix}
\qquad
\zeta_5=\begin{pmatrix}
0&0&0&0&1&0&0&0\\
0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&1&0\\
0&0&0&0&0&0&0&1\\
-1&0&0&0&0&0&0&0\\
0&-1&0&0&0&0&0&0\\
0&0&-1&0&0&0&0&0\\
0&0&0&-1&0&0&0&0
\end{pmatrix}
$$

$$
\zeta_2=\begin{pmatrix}
0&1&0&0&0&0&0&0\\
-1&0&0&0&0&0&0&0\\
0&0&0&-1&0&0&0&0\\
0&0&1&0&0&0&0&0\\
0&0&0&0&0&-1&0&0\\
0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&-1&0
\end{pmatrix}
\qquad
\zeta_6=\begin{pmatrix}
0&0&0&0&0&1&0&0\\
0&0&0&0&-1&0&0&0\\
0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&-1&0\\
0&1&0&0&0&0&0&0\\
-1&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0\\
0&0&-1&0&0&0&0&0
\end{pmatrix}
$$

$$\zeta_3 = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \qquad \zeta_7 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\zeta_4 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \qquad \zeta_8 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\delta_{km} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \qquad (2.13.33)$$

2.14 Conclusion on Chapter 2

In this paper, supported by a 16-sheeted atlas of metric spaces with sixteen types of signatures (topologies) (2.8.12) and a 32-page set of affine subspaces with stignatures (2.13.21), we obtain the metric-dynamic models of virtually all elements of the Standard Model.

Not considered in this article were the analogues of neutrinos, muons, tau-leptons and Higgs bosons. Metric-dynamic models of these vacuum formations (except for the Higgs boson), and the interaction between between them are shown in Chapters 3 through 8.

In the proposed here massless stochastic metaphysics there is no concept of "mass", so there is no need to introduce ideas about the

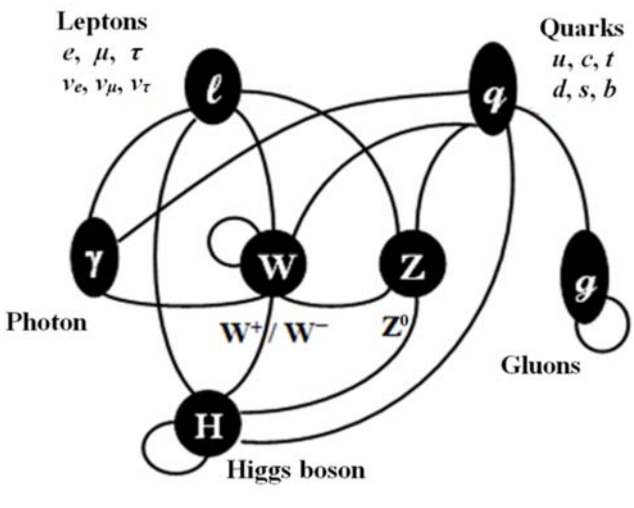


Fig 2.14.1 Components of the Standard Model

field, which provides a mechanism for spontaneous violation of electroweak symmetry, and, accordingly, about the quantum of this field – Higgs bosons. However, it is possible that in the fully geometrized theory there will be metric-dynamic models of vacuum formation with characteristics similar to those of these bosons.

Geometrized description of the main force interactions: electrostatic, electromagnetic, weak and nuclear will be presented in the following chapters.

Mathematical methods that allow to extract various information about local vacuum formations from the set of solutions of Einstein's vacuum equations are given at the beginning of this Chapter, and in Chapters 5 through 9 and [5, 22].

An article [32] shows that the Yang - Mills equations in four-dimensional space with conformal connection torsion reduce to Einstein's equations, Maxwell's equations, and another group of 10 of second-order differential equations. Another article by the same authors [33] provides a general solution to these equations for a centrally symmetric metric in the absence of an electromagnetic field, and also shows that among particular solutions of these equations, expressed in terms of elementary functions, there is a solution which is a Kottler metric.

In this work Kottler solutions are at the heart of model representations of the metric-dynamic vacuum organization as a whole given in (2.5.4) through (2.5.13), including the local spherical vacuum formations such as (2.6.22), (2.6.32) and (2.12.1). Therefore, the framework of the Algebra of Signatures provides a complete metric-dynamic «quark» model (Table 2.12.1) and practically all analogues of fermions and bosons (section 2.13.1 through 2.13.4) included in the Standard Model are also included in this framework, in line with the conclusions of [33]. These may then be proposed as a set of analytical solutions of the Yang-Mills theory.

Note that, if a set of metrics form (2.6.22) (2.6.32) and (2.12.1), then instead of:

$r_5 \sim 4.9\cdot10^{-3}$ cm: ~ «biological cage» inner core;

$r_6 \sim 1.7\cdot10^{-13}$ cm: ~ core of an elementary «particle»;

$r_7 \sim 5.8\cdot10^{-24}$ cm: ~ core of an «protoquark»;

we could substitute, for example,

$r_2 \sim 1.2\cdot10^{29}$ cm: ~ «metagalaxy» inner core;

$r_3 \sim 4\cdot10^{18}$ cm: ~ «galaxy» inner core;

$r_4 \sim 1.4\cdot10^{8}$ cm: ~ «star» or «planet» inner core,

continuing in an analogous manner, we obtain a geometrophysics and a topological description of the extent of the vacuum also on astronomical scales.

It appears to the author that this results in a universal metric-dynamic model of the closed universe which is, at the same time, on the average Ricci-flat; this universe is then populated by an infinite number of spherical vacuum formations of various sizes.

The usual probabilistic formalism of the Standard Model is still valid, as the core and “particelles” are stable vacuum formations constantly and randomly moving under the influence of the neighboring stable vacuum formations and a variety of other vacuum fluctuations. The study of the chaotic motion of the vacuum formation cores led to the conclusion of the Schrödinger equation (see Chapter 3), and Chapter 4 shows the connection of the Algebra of Signatures with quantum theories.

The Algebra of Signatures proposed in this article is not an alternative theory opposed to general relativity, quantum field theory and superstring theory, but rather their symbiosis via a full geometrization of physical laws.